Globalization and Labor Market Outcomes: 
Wage Bargaining, Search Frictions, and Firm Heterogeneity*

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Abstract

We introduce search unemployment into Melitz’ (2003) model of trade with heterogeneous 
firms. Firms have market power on product markets which leads to strategic wage bargaining. 
We analytically solve for the symmetric equilibrium. We find that the selection effect of trade 
influences labor market outcomes. Trade liberalization lowers unemployment and raises real 
wages as long as it improves average productivity net of transport costs. We show that this 
condition is likely to be met by a reduction in variable trade costs or the entry of new trading 
countries. On the other hand, the gains from a reduction in fixed market access costs are 
more elusive. Calibrating the model shows that the long-run impact of trade openness on 
the rate of unemployment is negative and quantitatively significant.

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1 Introduction

Public opinion meets globalization with mixed feelings. People agree that consumers benefit from trade but they are at the same time deeply concerned by its impact on job security. Fueled by numerous headlines about layoffs and outsourcing, many fear that globalization will worsen their prospects on the labor market.\(^1\) To a certain extent, economic theory can rationalize this fear. Workers who lose their jobs due to trade liberalization have to go through a period of active search before finding new employment opportunities. During this transition period, job reallocations increase the amount of frictions in the labor market which mechanically pushes up the rate of unemployment. On the other hand, comparatively little is known about the long-run effect of trade liberalization on unemployment. This is largely because equilibrium theories of trade and labor are still poorly integrated. In this paper, we attempt to bridge the two literatures by proposing a framework which combines the currently dominant approaches in each field.

We integrate a version of Melitz’s (2003) trade model with Pissarides’ (2000) canonical model of equilibrium unemployment. Building on Hopenhayn (1992) and Krugman (1980), the Melitz-model shows how trade liberalization affects the productivity distribution of firms through selection of efficient firms into exporting and of inefficient firms into exit. That selection effect enjoys massive empirical support\(^2\) and constitutes a tangible source of gains from trade that the earlier literature has paid little attention to. Our analysis suggests that it also matters for labor market outcomes. We find that, for reasonable parameter values, the cleansing effect of trade lowers search unemployment. As the cost of vacancy posting relative to the productivity of the average firm decreases, employers intensify their recruitment efforts. This raises the ratio of job vacancies to unemployed workers, which leads to lower unemployment and higher real wages.

Our framework modifies Melitz’s and Pissarides’ setups as follows. First, we neutralize the external scale effect that is inherent to the usual CES description of utility. This allows to concentrate on the selection effect that is novel to Melitz and avoids that the model features a negative correlation between country size and the equilibrium rate of unemployment, which would be at odds with empirical evidence. In the Appendix, we show that our results are robust to allowing for the existence of the scale effect.

\(^1\)Scheve and Slaughter (2001) provide a detailed analysis of how American workers perceive globalization.

\(^2\)See, among others, the surveys by Helpman (2006) or Bernard et al. (2007).
We also need to adapt the search-matching framework, which builds on competitive product markets, so as to make it compatible with the assumption of monopolistic competition used in trade models of the Krugman (1980) tradition. Allowing for monopoly power on product markets implies that we have to abandon matches as our unit of analysis and consider instead multiple-worker firms. Given the existence of search frictions, this introduces the complication of intra-firm bargaining. We focus on individual bargaining, where each worker is treated as the marginal worker and which is closest to competitive wage setting. However, in the Appendix to this article, we show that our main results hold in a collective setting where management bargains with firm-level unions about wages and employment.

Although the model features firms with heterogeneous productivity, monopoly power on product markets, external economies of scale, and, due to search frictions, monopsony power on labor markets, we are able to characterize its equilibrium in closed-form. The aggregation procedure proposed by Melitz goes through with little modification because, regardless of the bargaining environment, firms with different productivity levels pay similar wages. We also obtain a useful separability result according to which the equilibrium average productivity of input producers is independent from labor market outcomes. As a result, the system of equilibrium conditions turns out to be recursive. One can follow the same steps as Melitz (2003) to compute the average productivity in the economy and then solve for the equilibrium in the labor market.

The labor market equilibrium can be derived as in the standard Mortensen-Pissarides model by interacting a job creation and a wage curve. Then, whether trade liberalization improves or worsens labor market outcomes depends solely on how it affects average productivity. Even though trade liberalization reallocates market shares towards efficient firms, exporters also incur transport costs that have to be deducted from the productivity gains. This is why trade liberalization does not necessarily enhance average productivity net of transport costs. We establish that both average productivity and employment always increase following a reduction in variable trade costs or an increase in the number of trade partners, as long as fixed foreign distribution costs are larger than domestic ones. Given that this requirement is satisfied by realistic calibrations of the model, such liberalization policies are likely to improve labor market outcomes. The gains of reducing fixed costs for foreign firms turn out to be more elusive because such a change benefits almost exclusively to new exporters.\(^3\)

\(^3\) Introducing external economies of scale drives a wedge between average and aggregate productivity. It complicates the analysis as we also have to take into account the positive relationship between input diversity and average productivity. This new effect gives rise to an additional equilibrium relation and restricts the parameter space where the model admits a unique equilibrium. Setting aside these technical results, we find that economies
We conclude our analysis by a calibration exercise. Simulating various trade liberalization scenarios allows us to sort out the ambiguities, in particular regarding the role of fixed foreign costs, and to assess the magnitude of the effects. The simulations predict that reducing variable trade costs, or increasing the number of trade partners, has a significantly positive impact on both wages and employment.

**Related literature.** We build on our earlier work (Felbermayr and Prat, 2009) where we introduced search unemployment into a closed economy version of Melitz (2003) with the aim to study product market regulation. The relation to the present paper is straightforward, since trade liberalization can be understood as an alternative type of product market reform. In modeling bargaining regimes, we draw on Ebell and Haefke (2009), who analyze a closed-economy, *homogeneous* firms model of search and unemployment.

There is a growing number of theory papers on the trade-unemployment relationship with heterogeneous firms. Our approach is closely related to the recent work of Egger and Kreiche-meier (2009), who study the effect of trade liberalization in a model with *fair wages* and without search frictions. They find that trade increases the wage dispersion among identical workers and also leads to more unemployment. Davis and Harrigan (2007) find similar results for the degree of wage dispersion and unemployment, using an efficiency wages approach instead of fair wages.

There is a small but important literature on *search unemployment* in Heckscher-Ohlin trade models which goes back to Davidson et al. (1988). Building on their seminal line of research, three recent papers also discuss search unemployment in trade models with heterogeneous firms. The model closest to ours is presented by Janiak (2006). His framework exhibits an equilibrium under the assumption that the elasticity of substitution is smaller than two. As explained below, this restriction explains why Janiak’s model predicts that trade liberalization raises equilibrium unemployment. In our model, equilibrium existence and uniqueness is guaranteed under less restrictive and more plausible conditions.

Mitra and Ranjan (2007) and Helpman and Itskhoki (2007) introduce search unemployment in two-sector models with heterogeneous firms. Their papers differ from ours in terms of motivation and setup: Mitra and Ranjan discuss the role of off-shoring; Helpman and Itskhoki focus on how labor market distortions diffuse internationally through trade. When, as in our setup, countries are symmetric, the model of Helpman and Itskhoki features a negative trade-of scale do not modify the qualitative implications of the model. They actually reinforce the positive impact of trade liberalization by adding the variety-enhancing effect described in Krugman (1980) to the selection effect.
unemployment link: trade boosts average productivity in the differentiated goods sector, making employment there more attractive. This leads to a reallocation of labor from the distortion-free numéraire sector into the friction-ridden differentiated goods sector. Helpman, Itskoki and Redding (2008a, 2008b) propose models with heterogeneous firms and search frictions to address the effect of trade liberalization on wage inequality. There is also an emerging empirical literature on the effects of trade liberalization on aggregate unemployment. The cross-country evidence presented by Dutt, Mitra, and Ranjan (2009) suggests that higher openness improves labor market outcomes in the macroeconomy.

**Structure of the paper.** The remainder of the paper is organized as follows. Section 2 lays out the setup of the model. Section 3, solves for labor market equilibrium as a function of average productivity. In section 4, we show how firms’ exit and entry decisions shape average productivity in an economy open to international trade. Section 5 studies the effects of three globalization scenarios: (i) a reduction in variable trade costs, (ii) an increase in the number of trade relations, (iii) a reduction in fixed exporting costs. Section 6 calibrates the model in order to quantify the magnitude of the effects. Section 7 concludes. Proofs of the propositions, lemmata and corollaries are included in the Appendix.

## 2 Setup of the Model

We consider an economy that is essentially similar to the one analyzed in Melitz (2003) but for the existence of search frictions in the labor market. As in Melitz, the world is modeled as a collection of symmetric countries which interact on product markets. We deviate from existing treatments by neutralizing the external effect of input diversity on average productivity.

**Final output producers.** The setup of the production side of our model is akin to Egger and Kreickemeier (2009). The single final output good, $Y$, is produced under conditions of perfect competition and can be either consumed or used as an input in the production process. Good $Y$ is assembled from a continuum of intermediate inputs, which may be produced domestically or imported, and which may command different equilibrium prices. Denoting the quantity of

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4Davidson et al. (2007) propose a model with two-sided heterogeneity, where goods markets are perfectly competitive and firms endogenously choose technologies.

5For brevity, we skip the special case of autarky. Due to symmetry, we do not use country indices.
such an input \( q(\omega) \), we posit the following production function

\[
Y = \left[ M^{\nu-1} \int_{\omega \in \Omega} q(\omega)^{\sigma-1} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \ \nu \in [0, 1], \tag{1}
\]

where the measure of the set \( \Omega \) is the mass \( M \) of available intermediate inputs, each produced by a monopolistically competitive firm. We refer to \( M \) as the degree of input diversity while \( \sigma \) denotes the elasticity of substitution between any two varieties of inputs.

To understand the role played by \( \nu \), suppose that all varieties are demanded in identical quantities. Substituting \( q(\omega) = Q/M \), where \( Q \) is an aggregate index of input demand, yields \( Y = M^{\frac{\sigma-1}{\sigma}} Q \). If \( \nu = 0 \), then \( Y = Q \) and the number of available varieties is irrelevant for total output. This is the case discussed by Giavazzi and Blanchard (2003) or Egger and Kreickemeier (2009).\(^6\) If \( \nu = 1 \), the production function takes the conventional Dixit-Stiglitz form, where an increased number of varieties increases total output.

In the following, we set \( \nu = 0 \). This avoids a counterfactual negative correlation between the unemployment rate in autarky and the labor supply. With trade and symmetric countries, this counterfactual implication is maintained on the world level.\(^7\) We nonetheless allow for \( \nu > 0 \) in the Appendix to accommodate the dominant practice in the trade literature where gains from increased diversity are generally deemed important.

Setting \( \nu = 0 \), the price index dual to (1) is \( P = \left[ M^{-1} \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \), where \( p(\omega) \) is the price of input \( \omega \), inclusive of potential trade costs. We choose the final output good as the numéraire, i.e. \( P = 1 \). Then the demand of intermediate inputs \( \omega \) reads

\[
q(\omega) = \frac{Y}{M} p(\omega)^{-\sigma}. \tag{2}
\]

Intermediate input producers. At the intermediate inputs level, there is a continuum of monopolistically competitive firms which produce each a unique variety. Labor is the unique factor of production. It is inelastically supplied by the household and enters firms’ production functions linearly. Firms have different productivity levels \( \varphi(\omega) \), so that output \( q(\omega) = l(\omega) \varphi(\omega) \).

In the following, we use \( \varphi \) to index intermediate input producers.

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\(^6\)Our formulation of the aggregate production is formally similar to the utility function employed by Corsetti, Martin, and Pesenti (2007) who also stress the role of \( \nu \). Egger and Kreickemeier (2009) allow for \( \nu \in [0, 1] \) in the Appendix of their paper. Benassy (1996) discusses how the welfare properties of the Krugman (1980) model depend on \( \nu \). In particular, if \( \nu \neq 1 \), the decentralized equilibrium may yield over- or under-supply of input variety. This discussion carries over to the Melitz (2003) model.

\(^7\)With heterogeneous countries and costly trade, larger countries suffer less from trade costs, have a higher level of average productivity, and a lower rate of unemployment.
On the domestic and on each of the \( n \) symmetric export markets, input producers face fixed market access costs (e.g., distribution costs), \( f_D \) and \( f_X \) respectively.\(^8\) Throughout the paper, we assume that \( \tau^{\sigma-1} f_X > f_D \). As explained below, this ensures that only a subset of firms export and that exporters are on average more efficient than non-exporting firms, a well-established stylized fact in the trade literature (see for example the survey by Bernard et al. (2007)).

International trade is subject to variable iceberg trade costs \( \tau \geq 1 \) so that, in order to deliver a unit of input to a foreign market, the firm has to manufacture \( \tau \) units. If it decides to serve both the domestic and the foreign markets, a firm allocates its output so as to maximize its total revenues. Operating revenues from sales on a given foreign market are therefore equal to \( p_X q_X / \tau \).\(^9\) By symmetry, demands on the domestic and foreign markets are given by equation (2). Equating marginal revenues across markets therefore yields \( p_X(\varphi) = \tau p_D(\varphi) \) and \( q_X(\varphi) = \tau^{1-\sigma} q_D(\varphi) \), where \( D \) and \( X \) denote the domestic and the export market. Hence, total revenues are given by

\[
R(l; \varphi) \equiv \left[ \frac{Y}{M} \left( 1 + I(\varphi)n\tau^{1-\sigma} \right) \right]^{1/\sigma} (\varphi l)^{\frac{\sigma-1}{\sigma}},
\]

with \( I(\varphi) \) being an indicator function that takes value one when a \( \varphi \)-firm exports and zero otherwise. Apart from the fact that their effective demand level is multiplied by \( 1 + n\tau^{1-\sigma} \), exporting firms have similar revenue functions than non-exporting firms.

In order to facilitate the aggregation procedure, we define the average productivity level \( \tilde{\varphi} \) such that \( q_D(\tilde{\varphi}) = Y/M \). Hence, domestic sales of the average firm are equal to average sales per firm, and the domestic price of its good \( p_D(\tilde{\varphi}) = P = 1 \).

**Search frictions.** The labor market is imperfectly competitive due to the existence of search frictions. Whereas marginal recruitment costs are increasing at the aggregate level because of congestion externalities, they are exogenous from a firm’s point of view. The aggregate matching function is homogeneous of degree one so that the vacancy-unemployment ratio \( \theta \) uniquely determines the rate \( m(\theta) \) at which firms fill their vacancies. That rate is a decreasing function of \( \theta \) and satisfies the following standard properties: \( \lim_{\theta \to \infty} m(\theta) = 0 \) and \( \lim_{\theta \to 0} m(\theta) = \infty \). Due to the linear homogeneity of the matching function, job seekers meet firms at the rate \( \theta m(\theta) \) which is increasing in \( \theta \). The cost of posting vacancies is proportional to the parameter \( c \), so

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\(^8\)Since capital markets are perfect and uncertainty is resolved before market access costs are paid, \( f_X \) and \( f_D \) can be thought as flow fixed costs or – appropriately discounted – as upfront investment. In the latter case, whenever applicable, we use upper-case letters.

\(^9\)Notice that \( p_X \) is the c.i.f. price in the foreign market.
that recruiting $l$ workers entails spending \([c/m(\theta)]l\). In other words, firms face an adjustment cost function that is linear in labor.

3 Bargaining, wages, and unemployment

This section characterizes the labor market outcomes for given average productivity when wages are bargained individually. It shows that wages are constant across firms and the vacancy-unemployment ratio is increasing in average productivity. We devise our model in discrete time. All payments are made at the end of each period. Before the beginning of the next period, firms and workers are hit by idiosyncratic shocks: (i) with probability $\delta$, intermediate producers are forced to leave the market; (ii) with probability $\chi$, each job is destroyed because of match-specific shocks. We assume that these two shocks are independent so that $s = \delta + \chi - \delta\chi$ denotes the actual rate of job separation.

Unemployed workers earn a flow income $b\bar{w}$, where $b \in (0, 1)$, and which we index to the average wage rate in the economy, $\bar{w}$. Alternatively, one could also index the value of non-market activity to average productivity $\bar{\varphi}$ or to the final output good (whose price is normalized to unity). In the case where $\nu = 0$, the choice of indexation makes no important difference. On the other hand, when there are economies of scales ($\nu > 0$), indexation to the final output good leads to multiple equilibria, while the other normalizations ensure the existence of a unique equilibrium. For the sake of realism and in order to rule out multiple equilibria, we therefore choose the first option and denote the flow income of the unemployed as $b\bar{w}$.

3.1 Optimal vacancy posting

Individual wage bargaining involves the following sequence of actions: at each period, the intermediate input producer decides about the optimal number of vacancies $v$, taking the wage rate as given. The matching technology brings together the workers and the firm. Before production takes place, wages are bargained. Wage contracts are unenforceable: at any point in time, the firm may fire any employee and symmetrically any employee may quit. Solving the game by backward induction, we first characterize the firm’s optimal vacancy setting behavior, and then solve the bargaining problem.

\footnote{The implications of the linearity assumption are discussed below in footnote 16.}
The market value of an intermediate producer solves

\[
J(l; \varphi) = \max_v \left\{ \frac{1}{1+r} \left\{ R(l; \varphi) - w(l; \varphi) l - cv - f_D - I(\varphi) nf_X + (1 - \delta) J(l'; \varphi) \right\} \right\}, \quad (4)
\]

s.t. (i) \[ R(l; \varphi) = \left[ \frac{Y}{M^{v-1}} (1 + I(\varphi) n \tau^{1-\sigma}) \right]^{1/\sigma} (\varphi l)^{s-1} \sigma, \]

(ii) \[ l' = (1 - \chi) l + m(\theta) v, \]

where \( l' \) is the level of employment next period, and the dependence of \( l, v \) and \( q \) on \( \varphi \) is understood. Constraint (i) is the revenue function (3) and (ii) gives the law of motion of employment at the firm level. The first order condition for vacancy posting reads

\[
\frac{c}{m(\theta)} = (1 - \delta) \frac{\partial J(l', \varphi)}{\partial l'}, \quad (5)
\]

so that the firm sets the shadow value of labor equal to the expected marginal recruitment cost. Substituting the constraints into the objective function of the firm, differentiating with respect to \( l \), and using the optimality condition (5) yields

\[
\frac{\partial J(l, \varphi)}{\partial l} = \frac{1}{1+r} \left[ \frac{\partial R(l; \varphi)}{\partial l} - w(l, \varphi) - \frac{\partial w(l, \varphi)}{\partial l} l + \frac{c}{m(\theta)} (1 - \chi) \right]. \quad (6)
\]

The firm acts as a monopsonist by taking into account the effect of additional employment on the wage of inframarginal employees. The first order condition (6) regulates the optimal vacancy posting behavior of the firm, and hence, through the law of motion of employment, the optimal level of output. This, in turn, pins down the price of the intermediate input good: replacing the first order condition (5) on the left-hand side of (6) yields an expression that implicitly determines the optimal pricing behavior of the firm

\[
\frac{\partial R(l; \varphi)}{\partial l} = w(l, \varphi) + \frac{\partial w(l, \varphi)}{\partial l} l + \frac{c}{m(\theta)} \left( \frac{r + s}{1 - \delta} \right). \quad (7)
\]

This expression differs from the pricing rule considered by Melitz (2003) in that marginal costs are augmented by a monopsony effect \((\partial w(l, \varphi)/\partial l)\) and expected recruitment costs \(c(r + s)/(m(\theta)(1 - \delta))\).

3.2 Individual wage bargaining

The total surplus accruing from a successful match is split between the employee and the firm. The worker’s surplus is equal to the difference between the value of being employed \( E(l; \varphi) \) by a firm with productivity \( \varphi \) and workforce \( l \) and the value of being unemployed \( U \). The
firm’s surplus is simply equal to the marginal increase in the firm’s value \( \partial J (l; \varphi) / \partial l \) because individual bargaining implies that each employee is treated as the marginal worker. Following Stole and Zwiebel (1996) we assume that the outcome of bargaining over the division of the total surplus from the match satisfies the following “surplus-splitting” rule

\[
(1 - \beta) [E (l; \varphi) - U] = \beta \frac{\partial J (l; \varphi)}{\partial l}, \tag{8}
\]

where the parameter \( \beta \) measures the bargaining power of the worker and thus belongs to \([0, 1)\).  

As explained by Stole and Zwiebel (1996), condition (8) can be micro-founded either by cooperative or non-cooperative game theory. In the non-cooperative case, condition (8) characterizes the unique subgame perfect equilibrium of an extensive form game where the firm and its employees play the bargaining game of Binmore et al. (1986) within each bargaining session. Accordingly, neither the firm nor any employee can improve their positions by renegotiating. In the cooperative case, condition (8) assigns to each party its Shapley value, that is the average, over all possible permutations, of each player contribution to possible coalitions ordered below him.  

When \( \beta \) differs from 1/2, condition (8) generalizes the symmetric Shapley value to situations where players are not treated identically.

3.3 Labor market outcomes for given average productivity

Reinserting the shadow value of labor (6) in the bargaining solution (8) leads to an ordinary differential equation in the wage rate. Combining its solution with the endogenous outside option of workers, \( rU(\theta) \), we obtain a first relation between the degree of labor market tightness and the wage rate. We call it the Wage (W) curve. It reflects how behavior of firms and workers interact in the presence of monopoly power on product markets, search costs, and individual wage bargaining. Reinserting the solution of the differential equation satisfied by wages in the demand function for intermediate goods (2) yields a second relation between labor market tightness and the wage rate. Since this curve represents the demand for labor as a function of the bargained wage, we will hereafter refer to it as the Labor Demand (LD) curve.

**Proposition 1** Under individual bargaining and without external economies of scale \((\nu = 0)\), the labor market admits a unique equilibrium such that wages are constant across firms. The

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11 The case of \( \beta = 1 \) leads to the break-down of the labor market as firms could not finance the posting of vacancies.

12 This interpretation is the one favored by Helpman and Itskhoki (2007).

13 The term commonly used in the search-matching literature for this relationship is Job Creation curve.
equilibrium wage, $w$, and vacancy-unemployment ratio, $\theta$, simultaneously satisfy the following Wage and Labor Demand conditions:

\[ W: \quad w = B \frac{c}{1 - \delta} \left[ \frac{r + s}{m(\theta)} + \theta \right] \quad (9) \]

\[ LD: \quad w = \left( \frac{\sigma - 1}{\sigma - \beta} \right) \bar{\varphi} - \frac{c}{m(\theta)} \left( \frac{r + s}{1 - \delta} \right) \quad (10) \]

where $B \equiv \frac{\beta}{1 - \beta} \frac{1}{1 - \delta}$ is a measure of the bargaining power of the worker.

The Labor Demand and Wage curves are illustrated in Figure 1. Note that our choice of numeraire implies that $w$ is the real wage. The Wage curve implies that wages depend only on average productivity so that workers are paid similarly across firms with different productivity levels. This somewhat surprising result extends to a dynamic setting the proof of Stole and Zwiebel (1996) that firms exploit their monopsony power until employees are paid their outside option. The equalization of wages across heterogeneous firms can also be understood by looking at equation (5). It implies that the shadow value of employment is identical across firms. In other words, firms with different productivity levels choose employment levels such that the additional value created by the marginal worker is the same. Taking this insight to the surplus splitting rule (8), it is obvious that firms with different productivities have nevertheless the same surplus. It follows that the value of employment $E(l; \varphi)$ and, hence, the wage rate must be identical across firms.\footnote{This is a fairly general result that does not rely on functional forms.} The Wage curve is increasing in $\theta$ because the outside option is augmented by the recruitment costs that the firm has to pay in order to replace the worker. Quite intuitively, the workers’ bargaining position is improving in the severity of labor market frictions.

The Labor Demand curve shows that $\theta$ is an increasing function of the wage rate because firms post more vacancies when wages are low. The expression also shows that the wage rate depends positively on the average firm’s productivity $\bar{\varphi}$, but any increase in $\bar{\varphi}$ has a less than proportional effect on the wage rate due to firms’ bargaining power. The second term shows that higher expected search costs reduce the wage rate as they lead to a lower surplus out of a filled vacancy.

Figure 1 illustrates the effect of an increase in average productivity $\bar{\varphi}$ on labor market tightness. The Labor Demand curve shifts upwards (from the solid to the dashed curve) because firms are on average more productive and search more intensively for workers. The flow value of
non-market activity is simply set equal to an exogenous constant depending on the replacement rate and the equilibrium wage rate. Since the Wage curve is not affected by a change in $\bar{\varphi}$, labor market tightness goes from $\theta^0$ to $\theta^1$ so that Corollary 1 follows immediately.

**Corollary 1** The vacancy-unemployment ratio $\theta$ is increasing in average productivity $\bar{\varphi}$.

The intuition for Corollary 1 is straightforward: as long as firms can appropriate some of the rents from a filled vacancy (i.e., if $\beta < 1$), the equilibrium wage increases less than proportionally with average productivity so that filled vacancies become more valuable. Firms intensify their recruitment effort until the increased congestion of the labor market brings back the value of posted vacancies down to zero.

It is instructive to consider two special cases. First, assume that the costs of vacancy posting $c$ are indexed to the real wage. If that is the case, the Wage curve becomes vertical at some fixed level of $\theta$. The reason is that the workers’ outside option as well as their ability to extract rents does not change relative to the wage rate and hence bargaining settles at an unchanged employment level. Then, variations in $\bar{\varphi}$ is entirely absorbed by variations in the wage while the rate of unemployment does not change. If $c$ is at least partly indexed to the final output good,
unemployment is still affected by \( \tilde{\varphi} \). Second, assume that workers have no bargaining power, i.e., \( \beta = 0 \). Then, the Wage curve becomes horizontal at \( w = 0 \) and variations in \( \tilde{\varphi} \) are entirely absorbed by changes in labor market tightness.

It may also be helpful to contrast the Wage and Labor Demand curves derived above with those obtained in more standard settings with homogeneous firms and perfect competition on product markets (as described in Chapter 1 of Pissarides (2000)). In this setup, average and all firm-level productivities coincide and are equal to the price of individual varieties of the final output good. The curves take the same slopes as in our model and the mechanisms that underly those curves are identical: the Wage curve is upward-sloping because a higher \( \theta \) improves the outside option of workers, and the Labor Demand curve is downward-sloping because firms restrict the creation of vacancies when wages are higher.

In our model with heterogeneous firms, goods markets are more complex: the size of the surplus at the average firm depends on that firm’s productivity level instead of an exogenous price. Even in the absence of technological progress, average productivity can change when employment shifts between firms with different productivity levels. Additionally, firm-level prices and the aggregate price index do not coincide; this allows relative prices (absent in the canonical model) to affect recruitment decisions. In spite of these important differences, the similarity between our Wage and Labor Demand curves and those derived in the canonical search-matching model is striking. The standard model therefore turns out to be quite robust to introducing firm-level productivity differences, product differentiation, and monopoly power on goods markets.

To sum up, we have shown that, when \( c \) is not fully proportional to wages and \( \beta > 0 \), the rate of unemployment falls and the real wage rises when average productivity \( \tilde{\varphi} \) goes up. The next section explains how to endogenize \( \tilde{\varphi} \).

4 Firm Entry and Exit

We model firm entry and exit in a similar fashion than Melitz (2003), which in turn draws on the seminal work by Hopenhayn (1992). We deliberately keep the analysis as brief as possible and refer the reader to Melitz’ paper for further details. Our contribution is to show that the equilibrium level of average productivity \( \tilde{\varphi} \) and labor market tightness \( \theta \) are independent.

The entry process is in two stages. First, prospective entrants pay an entry cost \( F_E \). Only
after entering are they able to draw their productivity from a sampling distribution with c.d.f. \( G(\varphi) \) and p.d.f. \( g(\varphi) \). After the draw, productivities remain constant over time.\(^{15}\) Given that firms’ revenues are increasing in \( \varphi \), there exists a threshold \( \varphi^*_{D} \) below which firms do not take up production. Similarly, firms with a productivity level between \( \varphi^*_{D} \) and \( \varphi^*_{X} \) will serve only their domestic market. The share of exporting firms is therefore equal to \( \rho \equiv [1 - G(\varphi^*_{X})] / [1 - G(\varphi^*_{D})] \).

The average level of productivity of intermediate input producers is given by the following weighted sum

\[
\tilde{\varphi} = \frac{1}{1 + n\rho} \left[ \varphi^*_{D} g(\varphi^*_{D}) + n\rho \left( \frac{\varphi^*_{X}}{\tau} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}},
\]

(11)

where \( \tilde{\varphi}_{D} \) and \( \tilde{\varphi}_{X} \) are average productivity indices for the populations of firms that sell only domestically and that also sell abroad

\[
\tilde{\varphi}(\varphi^*_{D}) = \left[ \int_{\varphi^*_{D}}^{+\infty} \frac{\varphi^*_{D} g(\varphi)}{1 - G(\varphi^*_{D})} d\varphi \right]^\frac{1}{\sigma-1} \quad \text{and} \quad \tilde{\varphi}(\varphi^*_{X}) = \left[ \int_{\varphi^*_{X}}^{+\infty} \frac{\varphi^*_{X} g(\varphi)}{1 - G(\varphi^*_{X})} d\varphi \right]^\frac{1}{\sigma-1}.
\]

(12)

Because the adjustment cost function is linear in labor, firms reach their optimal size by the end of their first period of activity.\(^{16}\) It is therefore profitable to start operating and exporting when

\[
\frac{\Pi_i(\varphi)}{r + \delta} = \left( \frac{1 - \delta}{r + \delta} \right) \left[ p_i(\varphi) - w_i(\varphi) - \frac{c}{m(\theta)} \chi_i(\varphi) - f_i \right] - \frac{c}{m(\theta)} l_i(\varphi) - f_i \geq 0,
\]

(13)

where the subscript \( i \in \{D, X\} \) indicates whether the variables relate to domestic or foreign markets operations, and \( \Pi_i \) denotes expected flow profits net of recruitment costs. Condition (13) accounts for the fact that firms pay market access and vacancy costs upfront but have to wait one period to recruit their workers. In this period, they can be hit by a destruction shock, so that, with probability \( \delta \), they never start producing nor exporting. The cutoff productivities \( \varphi^*_i \) are such that the weak inequality in (13) binds.\(^{17}\)

\(^{15}\)This stylized assumption is made mainly for tractability reasons. It is the key difference between Melitz’s (2003) and Hopenhayn’s (1992) models, as the latter also allows firms’ productivities to vary over time.

\(^{16}\)Gradual convergence can be restored either by considering that recruitment costs are convex in the number of posted vacancies, as in Bertola and Caballero (1994), or by assuming that firms can post only one vacancy, as in Acemoglu and Hawkins (2007). Since this greatly complicates the aggregation procedure, we adopt a more stylized specification where, as in Melitz (2003), firms jump to their optimal size. See Koeniger and Prat (2007) for a numerical analysis of a model with firm entry and convex adjustment costs.

\(^{17}\)To see that some firms serve solely their domestic market, notice that \( R_X(\varphi) = p_X(\varphi) q_X(\varphi) / \tau = \tau^{1-\sigma} R_D(\varphi) \) and \( l_X(\varphi) = \tau^{1-\sigma} l_D(\varphi) \). Replacing these expressions in (13) shows that a \( \varphi^*_D \)-firm does not find it profitable to
The proportionality of domestic and foreign prices implies that expected profits of the marginal and average firms satisfy the following relation
\[
\Pi_i (\tilde{\phi}_i) + f_i = l_i (\tilde{\phi}_i) = \left( \frac{\tilde{\phi}_i}{\phi_i^*} \right)^{\sigma-1}, \text{ for } i \in \{D; X\}.
\]
Hence, (13) is equivalent to the following Zero Cutoff Profit (ZCP) conditions
\[
\Pi_i (\tilde{\phi}_i) = (1 + r) f_i \left[ \left( \frac{\tilde{\phi}_i}{\phi_i^*} \right)^{\sigma-1} - 1 \right], \text{ for } i \in \{D; X\}.
\]
Combining both ZCP conditions, and exploiting the relationship between domestic and export cutoff allows us to establish a single aggregate ZCP condition which only depends on the domestic cutoff
\[
\bar{\Pi} \equiv \Pi_D (\tilde{\phi}_D) + n_D \Pi_X (\tilde{\phi}_X) = (1 + r) \left[ f_D k(\varphi^*_D) + n_D k(\varphi^*_X) \right], \quad \text{(15)}
\]
where \( k(\varphi^*_D) = \left( \frac{\varphi^*_D}{\varphi^*_D} \right)^{\sigma-1} - 1, \) and \( k(\varphi^*_X) = \left( \frac{\varphi^*_X}{\varphi^*_X} \right)^{\sigma-1} - 1, \) are both implicitly pinned down by \( \varphi^*_D \) through the relationship between domestic and export cutoffs, and where \( \bar{\Pi} \) is average profits net of adjustment costs.\(^{18}\)

The ZCP conditions characterize the optimal decision of a firm who knows its idiosyncratic productivity. Imposing Free Entry (FE) allows us to take into account the behavior of prospective entrants. Entry occurs until expected profits are equal to the set up cost \( F_E \), so that
\[
\frac{F_E}{1 - G(\varphi^*_D)} = \bar{\Pi} = \frac{\bar{\Pi}}{r + \delta} \quad \text{(16)}
\]
Free entry holds when this equality is satisfied because an entrant will start to operate with probability \( 1 - G(\varphi^*_D) \) and then earn in each period an expected profit equal to \( \bar{\Pi} \).

Apart from the deduction of recruitment and churning costs from revenues in the definition of expected profits, the aggregate ZCP and FE conditions are the same as in Melitz (2003). It follows that existence and uniqueness of the equilibrium in the product market are ensured. More precisely, the aggregate ZCP decreases in \( \varphi^*_D \) when average profits tend to be higher as the firm that breaks even at \( \varphi^*_D \) becomes more productive.\(^{19}\) Conversely, the FE condition is incurred the exporting costs when, as assumed in Section 2, \( \tau^{\sigma-1} f_X > f_D \). When this partitioning does not hold, one cannot use equation (13) to determine whether or not a firm operates on the domestic market because it may be optimal to pay the fixed operating cost \( f_D \) in order to access the export markets.

\(^{18}\) Although \( \bar{\Pi} \) depends on both \( \varphi^*_D \) and \( \varphi^*_X \), it identifies the two variables because \( \varphi^*_X = \varphi^*_D \tau (f_X/f_D)^{\frac{1}{\sigma-1}} \). See the proof of Lemma 1 for a derivation of this equality.

\(^{19}\) Melitz (2003) shows that the ZCP is non-increasing in \( \varphi^*_D \). It can can well be horizontal, however, for example when firm productivities are sampled from a Pareto distribution.
upward-sloping: following an increase in $\phi^*_D$, a larger fraction of started firms will fail to draw sufficiently high productivity levels. This drives up the expected cost of successful entry so that FE requires higher expected profits.

The two equilibrium conditions (15) and (16) are independent of the vacancy-unemployment ratio $\theta$. Thus, as stated in the following Lemma, the entry and export thresholds depend solely on the product market parameters $\{F_E, f_D, f_X, n, \tau, r, \sigma, \delta\}$ and the properties of the c.d.f. $G(\phi)$.

Lemma 1 (Separability) The equilibrium average productivity of intermediate producers, $\tilde{\phi}$, does not depend on the vacancy-unemployment ratio $\theta$.

In other words, labor market conditions do not influence the selection of firms into failed ones, domestic sellers, and exporters. Separability holds because adjustment costs are linear in labor so that recruitment and churning expenses can be bundled with wages and treated as variable costs. Given that the endogenous variable, $\tilde{\Pi}$, is defined net of variables costs, the intensity of search frictions is immaterial to the analysis. In equilibrium, revenues will adjust so as to compensate changes in labor market tightness through opposite variations in the optimal sizes and number of firms. The neutrality of expenses that are proportional to the size of the labor force is already apparent in Melitz’s model since it is solved using the wages rate as numeraire. Lemma 1 shows that this feature continues to hold when variable costs include linear search costs on top of wages.

The separability property stated in Lemma 1 allows to solve for equilibrium in a recursive way. Average productivity and cutoff productivities can be determined as in Melitz (2003) by considering solely product market parameters. Taking these values as given, we can then solve for the equilibrium in the labor market. Note, however, that we would still need to determine input diversity $M$ in order to derive average productivity $\Phi$ if we would allow for the more sophisticated scenario with external economies of scale. As shown in the Appendix, external economies of scale lead to the introduction of an additional equilibrium condition.

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20 See equation (23) in the Appendix.
21 A related neutrality result can also be found in Bernard, Redding and Schott (2007). They assume that output, fixed and entry costs require using skilled and unskilled labor with common intensity. They show that factor rewards cancel out from the FE and ZCP conditions because average firm profitability and entry costs are each proportional to factor costs.
5 Unemployment and Trade Liberalization

This section discusses three globalization scenarios: (i) a reduction of variable trade costs, (ii) an increase in the number of trade relations and (iii) a drop in the fixed foreign distribution costs $f_X$. The first and the third scenario capture technological (transportation costs) and political (tariffs, technical barriers to trade) change, while the second addresses the emergence of new countries into the global trading system. We describe the interaction of trade liberalization and unemployment in two steps. First, we consider the case where trade affects aggregate outcomes through the selection effect only ($\nu = 0$). This isolates the novel mechanism introduced by Melitz (2003) and characterizes a particularly tractable special case. In the Appendix we analyze the more intricate case where trade also affects outcomes through an external scale effect, as in Krugman (1980) and in much of the subsequent literature.

5.1 The equilibrium rate of unemployment and the mass of firms

The steady-state rate of unemployment is linked to the degree of labor market tightness $\theta$ and the importance of labor market churning, as captured by $s$, via the standard Beveridge curve

$$u(\theta) = \frac{s}{s + \theta m(\theta)}.$$  \hspace{1cm} (17)

This condition ensures that the flows in and out of the unemployment pool are equal. As in standard search-matching models, the rate of unemployment is a decreasing function of the vacancy-unemployment ratio. Since we have shown in Propositions 1 and 2 that $\theta$ is increasing in the level of average productivity, it is sufficient to know how trade affects $\tilde{\varphi}$ in order to characterize its impact on employment.

The equilibrium mass of firms is obtained reinserting the equilibrium labor market tightness and the equilibrium levels of average productivity, as determined in sections 3.3 and 4, in the labor market clearing condition. Note that under our assumptions we have determined both $\theta$ and $\tilde{\varphi}_D$ without knowing the equilibrium mass of firms. Hence,

$$M_D [l_D(\tilde{\varphi}_D) + n_\theta l_X(\tilde{\varphi}_X)] = [1 - u(\theta)] L,$$  \hspace{1cm} (18)

where $L$ is the size of the labor force and $M_D$ is the mass of domestic producers in each country. Due to imports from foreign firms, input diversity $M$ (i.e., the number of available varieties) is higher and equal to $M = M_D (1 + n_\theta)$. 

16
As shown in the Appendix, the mass of firms is increasing in the level of employment. Although this may seem obvious, it turns out that, in the general case where \( \nu \geq 0 \), the relationship is actually ambiguous. This is because \( M \) has two opposite effects: (i) at the aggregate level, a larger number of firms naturally increases the number of employees; (ii) at the firm level, economies of scale imply that more input diversity raises revenues per worker so that firms have to be smaller for the ZCP condition to be satisfied. When \( \sigma > \nu + 1 \), which is an empirically plausible restriction, effect (i) dominates and the equilibrium mass of firms increases in employment.\(^{22}\)

5.2 The effect of trade liberalization on labor market outcomes

To ascertain the effect of trade on labor market outcomes, it is sufficient to see how it affects average productivity, \( \bar{\phi} \). As in Melitz (2003), trade liberalization moves the ZCP but does not affect the FE condition. Trade affects the distribution of productivities across intermediate input producers by reallocating labor towards exporters, which are situated at the upper tail of the productivity distribution, and away from purely domestic firms, both at the extensive and at the intensive margin. Nevertheless, the effect of trade liberalization on average productivity is ambiguous because \( \bar{\phi} \) factors in the output loss in export transit. Figure 2 illustrates a situation where the ZCP shifts right so that average productivity actually increases.

Part (i) of the following proposition gives a sufficient condition under which some liberalization scenarios always lead to an increase in aggregate employment. Part (ii) derives necessary and sufficient conditions for the case where the productivity distribution \( G(\phi) \) belongs to the Pareto family of distributions, as usually done in the literature on heterogeneous firms.\(^{23}\)

**Proposition 2** (i) If \( f_X \geq f_D \), a reduction of variable trade costs \( \tau \) or an increase in the number of trading partners \( n \) lead to a fall in the equilibrium rate of unemployment and a rise in the real wage, regardless of whether wages are bargained individually or collectively. A fall in fixed foreign distribution costs has an ambiguous effect on labor market outcomes.

---

\(^{22}\)When \( \sigma < \nu + 1 \), so that firms enjoy strong market power and external economies of scale are significant, \( M \) is decreasing in aggregate employment because the effect at the firm level dominates. This is the case studied by Janiak (2006). Such a parameter restriction is, however, in contradiction with empirical studies which typically yield estimates for \( \sigma \) above 2 and for \( \nu \) in the interval \((0, 1)\). Hence, we restrict our attention to cases where \( \sigma > \nu + 1 \). Egger and Kreickemeier (2009) also impose a similar restriction in order to ensure that their equilibrium is stable.

(ii) Let firms draw their productivities from a Pareto distribution with dispersion parameter \( \gamma \) such that \( \gamma > \sigma - 1 \). Then, regardless of the wage bargaining regime, the equilibrium rate of unemployment falls and the real wage rises
(a) due to a reduction in \( \tau \) or an increase in \( n \) if and only if
\[
\frac{\sigma - 1}{\gamma} \left( 1 + n\tau^{-\gamma} \left( \frac{f_D}{f_X} \right)^{\frac{2}{\sigma - 1}} \right) \geq \frac{f_D}{f_X} - 1,
\]
(b) and due to a reduction in \( f_X \) if and only if
\[
\frac{\sigma^2}{(\sigma - 1)^2} \geq \frac{f_X}{f_D} \left[ 1 + n\tau^{-\gamma} \left( \frac{f_D}{f_X} \right)^{\frac{2}{\sigma - 1}} \right].
\]

The new insight in Melitz (2003) is that trade liberalization reallocates market shares towards efficient firms. Exporters, however, also incur iceberg transport costs which have to be deducted from the productivity gains at the factory gate. Whether or not trade liberalization enhances average productivity depends on which of these two adjustments prevails.\(^{24}\) When \( f_X > f_D \), revenues generated on each foreign market have to exceed domestic revenues. In other words, the higher efficiency of exporting firms offsets both transport costs and the difference between

\(^{24}\) Melitz (2003) briefly alludes to the ambiguity of the relationship between trade liberalization and \( \phi \) (see footnote 26, page 1713). He also introduces a measure of productivity at the factory gate and shows that it is always lower in autarky.
$f_X$ and $f_D$. This is why the selection effect always dominates the losses in export transit. On the other hand, when $f_X < f_D$, some of the transport costs are compensated by lower fixed costs in foreign markets. Then the productivity gains at the factory gate due to trade liberalization are not necessarily higher than the increase in export losses.

A reduction in fixed costs of export $f_X$ triggers similar adjustments than a decrease in $\tau$: it raises the domestic threshold $\varphi^*_D$ and lowers the export threshold $\varphi^*_X$. Yet, it reallocates market shares in a different way. Whereas a decrease in $\tau$ raises the combined market shares of firms that already exported prior to liberalization, a decrease in $f_X$ mostly benefits new exporters which are, on average, less productive than existing ones. Hence, the overall effect on average productivity is ambiguous and depends on whether the new exporters are on average more productive than the economy-wide average before the fall in $f_X$.

The region where the relationship between trade openness and average productivity is negative depends on the other parameters of the model. It can be characterized when parametric assumptions are imposed on the sampling distribution $G(\varphi)$, as shown in part (ii) of Proposition 2 for cases where the sampling distribution is Pareto. Note that the effect of $f_X$ is non-linear, since the stated parameter restrictions depends on $f_X - f_D$. If that difference is negative, a reduction in fixed market access costs always lowers unemployment.

In section 6 we calibrate the model towards U.S. data. This allows to assess whether or not the conditions required for a beneficial impact of trade liberalization on labor market outcomes are likely to be met in reality.

6 Numerical Illustration

Although our theoretical model can be fully characterized analytically, the effect of trade liberalization on labor market outcomes is potentially ambiguous, a beneficial effect requiring specific restrictions on exogenous parameters. Calibrating the model illustrates that those restrictions are likely to hold in reality. We simulate the labor market effects of different trade liberalization scenarios to shed light on the quantitative importance of the trade-unemployment nexus. Our numerical exercise is merely illustrative since we model a world of perfectly symmetric countries. Also note that we focus on the long-run and neglect adjustment dynamics in the two key variables of the model, $\theta$ and $\tilde{\varphi}$.

Our calibration follows standard practice, as versions of the Melitz (2003) and of the Pis-
Sarides search-matching model have been frequently calibrated in the literature. Regarding the product market, we follow Bernard, Redding, and Schott (2007); calibration of the labor market side is close to Shimer (2005).

6.1 Calibration

In the following, we describe the calibration of our model. Table 1 summarizes all parameter values and statistics are for monthly values.

Sampling distribution and aggregate production function. As Bernard et al. (2007), Ghironi and Melitz (2005) or Helpman, Melitz, Yeaple (2004), we assume that firm productivities are distributed according to a Pareto distribution. Setting the scale parameter of that distribution to unity, the probability density is $g(\varphi) = \gamma \varphi^{-(1+\gamma)}$. The shape parameter $\gamma$ governs the rate of decay of the distribution. We need to impose $\gamma > \sigma - 1$ to ensure that the variance of the sales distribution is finite. As Bernard, Redding, and Schott (2007), we set $\gamma = 3.4$ and choose $\sigma = 3.8$.

Variable and fixed costs of trade and entry. We normalize the number of potential workers and set $L = 1$. We choose variable trade costs $\tau$ equal to 1.3 as Ghironi and Melitz (2005). Given the Pareto distribution, the share of firms that export is given by $\varrho = \tau^{-\gamma} \left( \frac{f_D}{f_X} \right)^{\frac{\gamma}{\sigma - 1}}$. That number is put at about 21% by Bernard et al. (2003). Together with $\tau = 1.3$, this pins down the ratio $f_X/f_D$ at about 1.7. Setting the number of trading partners $n = 2$, we obtain an overall degree of openness (export sales over total sales) of about 19%. Finally, we calibrate $F_E = 39.57$ and $f_D = 1.77$ such that the equilibrium labor market tightness produced by our model is 0.5 (Hall, 2005) and the average firm size is equal to 21.9 (Axtell, 2001).

\[25\text{See Felbermayr and Prat (2007) for a related calibration exercise for the case of a closed economy.}\]

\[26\text{Without external economies of scale, the size of the labor force is meaningless for the labor market outcomes.}\]

\[27\text{The relation between } F_E \text{ and } f_D \text{ is of the same order of magnitude than in Ghironi and Melitz (2005).}\]
Separation shocks. Job separations occur either because a firm exit the market or because the match itself is destroyed. Bartelsmann et al. (2004) estimates are centered around a monthly hazard rate of exiting the market $\delta = 0.96\%$. Match-specific shocks account for the job separations that are left unexplained by firm-specific shocks. Shimer (2005) estimates the monthly job separation rate to be on average equal to $s = 0.034$. It follows that the monthly Poisson arrival rate of match-specific shocks $\chi = \frac{s - \delta}{1 - \delta} \approx 0.024$.

Parameters for the matching function and cost of vacancy-posting. We postulate a Cobb-Douglas matching function $m(\theta) = m_0 \theta^{-\alpha}$, whose elasticity $\alpha$ is set equal to 0.5 following Petrongolo and Pissarides (2001). The assumption of constant returns to matching implies that $\theta$ is equal to the job finding rate $m(\theta)\theta$ over the job filling rate $m(\theta)$. Shimer (2005) estimates the monthly job finding rate in the U.S. to be around 0.45, whereas Hall (2005) finds an average labor market tightness $\theta$ of around 0.5. It follows that the monthly job filling rate $m(\theta)$ is equal to $0.45/0.5 \approx 0.9$, so that $m_0 \approx 0.64$. We target the flow income of unemployment to be 40% of the equilibrium real wage, which yields a value for $b$ of around 0.32. Firms’ vacancy posting costs are fixed to 1.1 times the monthly wage (Eberr and Haefke, 2006). We calibrate those costs at 4.73, which appears large compared to flow fixed costs.

Bargaining power and value of non-market activity. The results of Abowd and Allain (1996) suggest that, in the case of individual bargaining, workers’ bargaining power is close to $\beta = 0.5$.

6.2 The labor market effects of trade liberalization

The calibrated parameters summarized in Table 1 show that the sufficient condition, $f_X/f_D > 1$, for lower variable trade costs to reduce unemployment is very likely to be met. Foreign relative to domestic distribution costs need to be large for the model to be consistent the low export participation rates of firms. Moreover, the sufficient and necessary condition for foreign market access costs is met in the neighborhood of the calibrated value of $f_X$.\textsuperscript{28} Hence, from Table 1 it is possible to conclude that all three trade liberalization scenarios lead to lower equilibrium

\textsuperscript{28}Moreover, the values of $\nu, \sigma$, and $\alpha$ provided by the empirical literature suggest that the existence and uniqueness requirement of Lemma 2 is fairly weak.
Table 1: Calibration-Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>Bargaining power, individual bargaining</td>
<td>0.5</td>
<td>Abowd and Allain (1996)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Elasticity of matching function</td>
<td>0.5</td>
<td>Petrongolo and Pissarides (2001)</td>
</tr>
<tr>
<td>(s)</td>
<td>Monthly job destruction</td>
<td>3.4%</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>(r)</td>
<td>Monthly discount rate</td>
<td>0.33%</td>
<td>4% annual interest rate</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Monthly rate of firm exit</td>
<td>0.97%</td>
<td>Bartelsmann et al. (2004)</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Elasticity of substitution</td>
<td>3.8</td>
<td>Bernard et al. (2007)</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Decay of productivity distribution</td>
<td>3.4</td>
<td>Bernard et al. (2007)</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Iceberg trade costs</td>
<td>1.3</td>
<td>Ghironi &amp; Melitz (2005)</td>
</tr>
</tbody>
</table>

Parameters matched to moments in the data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta)</td>
<td>Entry costs</td>
<td>39.57</td>
<td>(\theta \approx 0.5) (Hall, 2005)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Domestic flow fixed costs</td>
<td>1.77</td>
<td>Average firm size = 21.9</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Fixed foreign market access costs</td>
<td>3.01</td>
<td>(\rho = 0.21) (Bernard et al., 2003)</td>
</tr>
</tbody>
</table>

Normalized Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L)</td>
<td>Labor endowment per country</td>
<td>1</td>
</tr>
<tr>
<td>(P)</td>
<td>Aggregate price level</td>
<td>1</td>
</tr>
<tr>
<td>(n+1)</td>
<td>Number of countries</td>
<td>3</td>
</tr>
</tbody>
</table>

Note: All parameter values and statistics are for monthly time periods and are calibrated towards the U.S. economy.
unemployment and higher real wages.

By simulating the model, we can go beyond these findings. First, while the theoretical analysis is local, our numerical exercise allows for a global analysis; this is particularly relevant for fixed costs of accessing foreign markets as they have non-linear effects. Second, by means of simulation we can quantify the unemployment-reducing effect of trade liberalization.

Figure 3 illustrates the effect of trade liberalization on labor market outcomes. The top diagram studies variations in variable trade costs \( \tau \), the middle diagram analyzes changes in the number of countries to which any country may export to \( n \), and the bottom diagram shows the effects of changing fixed costs of foreign market access. All pictures have the real wage on the right ordinate and the unemployment rate (in percent) on the left ordinate.\(^{29}\) The baseline calibration at \( \tau = 1.3, n = 2, f_X = 3.01 \) shows an unemployment rate of 7% and a wage rate of 4.3.

The first row of Figure 3 illustrates that lower variable trade costs \( \tau \) can have a sizable effect on labor market outcomes. In the case of individual bargaining, moving \( \tau \) from 1.6 to 1 lowers the unemployment rate by 1 percentage point from 7.4% to 6.4% and increases the wage rate from 3.9 to 5.2.\(^{30}\)

The second row in Figure 3 relates to variation in the number of export markets. In the baseline case we have \( n = 2 \). Now, consider an increase of \( n \) to, say, 4: the wage rate goes up to about 4.9 while the unemployment rate falls to 6.6%.

Finally, consider the case of a change in the fixed foreign market access cost \( f_X \). A marginal reduction in \( f_X \) at the baseline equilibrium \( (f_X = 3.01) \) leads to an increase in the unemployment rate and to a decrease in the real wage.\(^{31}\) When \( f_X \) decreases, input diversity goes up, but

\(^{29}\)Obviously, the intersection of both curves has no particular meaning.

\(^{30}\)The discussed reduction of \( \tau \) from 1.6 to 1 describes the entirely unrealistic transition from costly trade to a situation where no trade-costs whatsoever exist. Since higher \( \tau \) lowers the effective labor productivity, reducing \( \tau \) by 60% has a massive effect on average productivity. With \( n \) growing towards infinity, the share of imported inputs converges towards 1 and a reduction in \( \tau \) is equivalent to an increase in the marginal productivity of labor.

\(^{31}\)Notice that the firm’s productivity distribution is only lower bounded by \( \bar{\varphi} \). A rather strong increase in \( f_X \) leads to a lower domestic cutoff \( \varphi_D^\ast \) and higher export cutoff \( \varphi_X^\ast \). The economy moves towards autarky, which implies that the domestic cutoff converges to its autarky value. The export cutoff is not bounded and thus goes to infinity. However, the mass of remaining exporters becomes negligible small, so that their impact on average productivity is small and the reduction of average productivity in the domestic regime dominates after a certain threshold. The average productivity in the domestic regime becomes more important for the total average
average productivity ($\tilde{\varphi}$) falls if $f_X$ dips below some threshold. This is because the selection effect weakens: lower $f_X$ reduces the prices charged by incumbent exporters, but also allows less efficient firms to export. The net impact is ambiguous ex ante, and becomes negative if $f_X$ is low enough. Increasing $f_X$ beyond a certain threshold then leads to a situation where both effects net each other out. To illustrate these countervailing mechanisms, we allow for a rather large increase in foreign market access cost $f_X$ in the last row of Figure 3. Further increase of $f_X$ beyond 8 leads to higher unemployment and lower wages rate, as predicted by Proposition 2. However, the magnitude of the changes is rather small.

For the parameters used in our benchmark calibration Proposition 2 pins down the turning point at $f_X \approx 7.7$. 

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**Figure 3: Simulation results**

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7 Conclusion

Bringing together two important established but hitherto unrelated models in the trade and labor literatures – the Melitz (2003) model of trade with heterogeneous firms, and the Pissarides (2000) search and matching approach to unemployment – this paper develops conditions under which the selection effect of trade improves labor market outcomes. The proposed framework is surprisingly tractable, in spite of the existence of heterogeneous firms, various types of trade costs, monopoly power on product markets, and monopsony power due to search frictions on the labor market. The equilibrium is recursive since labor market conditions do not affect average productivity (the converse, of course, is not true). Introducing external economies of scale drives a wedge between average and aggregate productivity. Then, aggregate productivity does depend on labor market outcomes.

The paper shows that the labor market implications of trade liberalization are largely shaped by its impact on average productivity. This latter relation, however, depends on parameter constellations. To sort out the ambiguities, we calibrate the model towards U.S. data. We find that different trade liberalization scenarios all improve labor market outcomes, regardless of the bargaining environment. Moreover, the reduction in the unemployment rate is numerically non-trivial, in particular when wages are bargained individually and external economies of scale are important.

Compared to existing models that combine search unemployment and heterogeneous firms, our treatment features forward-looking firms, micro-founds the wage determination, and allows one to derive the main results without any assumptions on the distribution of firm productivities. External economies of scale are shown to be important for the model’s properties. Existence and uniqueness do not require strong assumptions on parameters, and the model is straightforwardly calibrated. There are, however, two obvious and interesting extensions which we have to relegate to future research.

First, our approach focuses on long-run equilibria. This precludes the analysis of potentially interesting short-run adjustments, which result from the fact that the mass of producers adjusts only sluggishly to a changed environment due to sunk entry costs. Most empirical studies on the interaction between trade liberalization and labor turnover capture short to medium-run correlations, so that our model has little to say about their results. Moreover, any sensible welfare analysis requires to weigh potential losses along the transition path against the positive long-run effects.
Second, our conclusions are limited to the impact of multilateral trade liberalization amongst symmetric countries. Hence, we cannot say much about the recent surge in bilateral trade treaties or, even more importantly, about the effect on employment of trade liberalization with emerging countries such as China or India. We therefore believe that the most promising direction for further research would be to extend the model to cases where countries differ with respect to sizes, productivity levels and institutions. This will probably be a rather demanding project since addressing country asymmetries has proved difficult in the literature, in particular if one is not willing to narrow the analysis to two countries or to allow for a numéraire sector whose output is costlessly tradable.
A Proofs

**Proof of Proposition 1.** To solve the surplus-splitting rule (8), notice that the optimality condition (5) does not vary with the level of the control variable $v$. Hence, the optimal firm size remains constant through time, so that $l = l'$. This condition and the envelope theorem enable us to rewrite (6) as

$$\frac{\partial J(l, \varphi)}{\partial l} = \left( \frac{1}{r + s} \right) \left[ \frac{\partial R(l; \varphi)}{\partial l} - w(l, \varphi) - \frac{\partial w(l, \varphi)}{\partial l} l \right].$$

Reinserting this expression together with $E(\varphi) - U = (w(l, \varphi) - rU)/(r + s)$ into (8) yields

$$w(l, \varphi) = \beta \frac{\partial R(l; \varphi)}{\partial l} + (1 - \beta)rU - \beta \frac{\partial w(l, \varphi)}{\partial l} l.$$  (19)

Equation (26) is a linear differential equation in $l$. One can verify by direct substitution that

$$w(l, \varphi) = (1 - \beta)rU + \beta \left( \frac{\sigma}{\sigma - \beta} \right) \frac{\partial R(l, \varphi)}{\partial l}.$$  (20)

solves (26). Equation (20) is the counterpart of the Wage curve in the standard search-matching model. The Labor Demand curve is derived reinserting the demand function (2) into (20) and differentiating the resulting equation with respect to $l$

$$\frac{\partial w(l, \varphi)}{\partial l} l = -\frac{1}{\sigma} \left[ \beta \left( \frac{\sigma}{\sigma - \beta} \right) \frac{\partial R(l; \varphi)}{\partial l} \right].$$

This expression allows us to substitute $(\partial w(l, \varphi)/\partial l) l$ in (7) to obtain

$$w(l, \varphi) = \left( \frac{\sigma}{\sigma - \beta} \right) \frac{\partial R(l; \varphi)}{\partial l} - \left( \frac{r + s}{1 - \delta} \right) \frac{c}{m(\theta)}.$$  (21)

Finally, we express the Wage Curve as a function of $\theta$ by reinserting (20) into (21)

$$w(l, \varphi) = rU + \left( \frac{\beta}{1 - \beta} \right) \left( \frac{r + s}{1 - \delta} \right) \frac{c}{m(\theta)}.$$  (22)

It follows that wages are identical across firms. Thus the workers’ outside option reads

$$rU(\theta) = b\bar{w} + \theta m(\theta) \left( \frac{w - rU}{r + s} \right) = b\bar{w} + \frac{\beta}{1 - \beta} \left( \frac{c\theta}{1 - \delta} \right),$$

where (22) is used to drop the dependence of $w$ on $l$ and $\varphi$. The Wage curve in Proposition 1 follows after reinserting the expression of $U$ into (22). To simplify the Labor Demand curve, consider first a firm

---

33 See Bertola and Garibaldi (2001) or Ebell and Haefke (2006) for a detailed solution of a similar ODE by the method of variation of parameters.
that does not export, so that \( I(\varphi) = 0 \). In this case, it is easily seen that the iso-elastic demand (2) implies \( \partial R(l; \varphi) / \partial l = p_D(l; \varphi) \varphi (\sigma - 1) / \sigma \). This equality also holds true for exporting firms because they are facing the same domestic demand than non-exporting firms and that marginal revenues are equalized across markets. To see this formally, notice that

\[
\frac{\partial R(l; \varphi)}{\partial l} = \left( \frac{\sigma - 1}{\sigma} \right) \left[ \frac{Y}{M^{1-\nu}} (1 + n r^{1-\sigma}) \right]^{1/\sigma} \varphi q(l; \varphi)^{-\frac{1}{\sigma}}
\]

\[
= \left( \frac{\sigma - 1}{\sigma} \right) \left[ \frac{Y}{M^{1-\nu}} \right]^{1/\sigma} q_D(l; \varphi)^{-\frac{1}{\sigma}} \varphi = \left( \frac{\sigma - 1}{\sigma} \right) p_D(l; \varphi) \varphi , \text{ when } I(\varphi) = 1.
\]

The second equality holds true because output is optimally allocated across markets when \( q_X = q_D r^{1-\sigma} \).

Equation (21) therefore implies that \( p_D (\varphi) \varphi = p_D (\tilde{\varphi}) \tilde{\varphi} = \tilde{\varphi} \), where the last equality follows from the definition of \( \tilde{\varphi} \). These simplifications lead to the Job Creation condition reported in Proposition 1.

The uniqueness of the equilibrium is ensured since the Wage curve is increasing in \( \theta \) and the Labor Demand curve decreasing. Existence from the same reason since the intercept of the Wage curve is smaller than that of the Labor Demand curve, which yields the condition stated in Proposition 1.

**Proof of Corollary 1.** Combining the Labor Demand and Wage curves in Proposition 1 leads to the following equilibrium requirement

\[
\Psi(\theta; \tilde{\varphi}) \equiv \tilde{\varphi} \left( -\frac{\sigma - 1}{\sigma - \beta} \right) + \frac{c}{(1-\beta)(1-b)} \left( (1-b(1-\beta))(r+s) + \beta \theta \right) = 0.
\]

Differentiating \( \Psi(\theta; \tilde{\varphi}) \) with respect to \( \tilde{\varphi} \) and \( \theta \) yields

\[
\frac{\partial \tilde{\varphi}}{\partial \tilde{\varphi}} = - \frac{\partial \Psi(\theta; \tilde{\varphi})/\partial \tilde{\varphi}}{\partial \theta} = - \frac{-\sigma - 1}{\sigma - \beta} \left( \frac{-c}{(1-\beta)(1-b)} \left( (1-b(1-\beta))(r+s) + \beta \theta \right) + \frac{\beta}{1-\delta} \right) > 0.
\]

**Proof of Lemma 1.** The Lemma follows from the two equilibrium conditions (15) and (16). Yet, we still have to establish that the relationship between \( \varphi_D \) and \( \varphi_X \) does not depend on \( \theta \). From the definition of the cutoff productivity in equation (13)

\[
\pi_X (\varphi_X^*) + f_X - \left( \frac{r + \delta}{1 - \delta} \right) \frac{c}{m(\theta)} l_X (\varphi_X^*) = r^{1-\sigma} \left[ \pi_D (\varphi_D^*) + f_D - \left( \frac{r + \delta}{1 - \delta} \right) \frac{c}{m(\theta)} l_D (\varphi_D^*) \right] = \left( \frac{r + \delta}{1 - \delta} \right) f_X.
\]

But we also know that employment levels are log-linear functions of \( \varphi \), so that

\[
\pi_D (\varphi_D^*) + f_D = \left( \frac{r + \delta}{1 - \delta} \right) \frac{c}{m(\theta)} l_D (\varphi_D^*) = \left( \frac{\varphi_D^*}{\varphi_D} \right)^{\sigma - 1} \left[ \pi_D (\varphi_D^*) + f_D - \left( \frac{r + \delta}{1 - \delta} \right) \frac{c}{m(\theta)} l_D (\varphi_D^*) \right] = \left( \frac{\varphi_D^*}{\varphi_D} \right)^{\sigma - 1} \left( \frac{r + \delta}{1 - \delta} \right) f_D,
\]

28
where the last equality follows from the definition of $\varphi_D^*$. Combining the two relations above, yields the same relationship than in Melitz (2003): $\varphi_X^* = \tau \varphi_D^* (f_X/f_D)^{\frac{1}{1-\tau}}$. This equation allows us to use (16) to pin down $\varphi_D^*$. We can then use (12) to express $\tilde{\varphi}_k$ as a function of $\varphi_k^*$, for $k \in \{D, X\}$.

**Equilibrium mass of firms.** We use the labor market clearing condition to derive the equilibrium mass of firms when wages are bargained at the individual level. The average levels of employment follow from the requirement that the profits of $\varphi_D^*$-firms be zero

$$l_i(\varphi_i^*) \left[ (\tilde{\varphi} - w) \frac{1 - \delta}{r + \delta} - \frac{c}{m(\theta)} \frac{r + s}{r + \delta} \right] = f_i \left( \frac{1 + r}{r + \delta} \right), \text{ for } i \in \{D, X\},$$

(23) and the log-linear relation between firm sizes: $l_i(\tilde{\varphi}) = (\tilde{\varphi}_i/\varphi_i^*)^{\sigma-1} l_i(\varphi_i^*)$, for $i \in \{D; X\}$. Reinserting the Labor Demand curve yields

$$l_i(\tilde{\varphi}_i) = \left( \frac{\tilde{\varphi}^*_i}{\varphi_i^*} \right)^{\sigma-1} \left( \frac{1 + r}{1 - \delta} \right) \left( \frac{\sigma - \beta}{1 - \beta} \right) \frac{f_i}{\varphi}, \text{ for } i \in \{D, X\}.$$

Accordingly, equation (18) is equivalent to

$$M_D = L \left( \frac{\theta m(\theta)}{s + \theta m(\theta)} \right) \left( 1 - \beta \right) \left( \frac{1 - \delta}{1 + r} \right) \left[ \left( \frac{\tilde{\varphi}_D}{\varphi_D} \right)^{\sigma-1} \left( \frac{f_D}{\varphi} \right) + n\varphi \left( \frac{\tilde{\varphi}_X}{\varphi_X} \right)^{\sigma-1} \left( \frac{f_X}{\varphi} \right) \right]^{-1},$$

Using the Free Entry condition (16), we can rearrange this expression as follows

$$M_D = \tilde{\varphi} L \left( \frac{\theta m(\theta)}{s + \theta m(\theta)} \right) \left( 1 - \beta \right) \left( \frac{1 - \delta}{1 + r} \right) \left[ \left( \frac{r + \delta}{1 + r} \right) \frac{f_X}{1 - G(\varphi_D^*)} + f_D + n\varphi f_X \right]^{-1}.$$

In order to get the final solution for the number of available varieties, one has to take $M_I = (1 + n\varphi)M_D$ into consideration, so that

$$M = (1 + n\varphi)\tilde{\varphi} L \left( \frac{\theta m(\theta)}{s + \theta m(\theta)} \right) \left( 1 - \beta \right) \left( \frac{1 - \delta}{1 + r} \right) \left[ \left( \frac{r + \delta}{1 + r} \right) \frac{f_X}{1 - G(\varphi_D^*)} + f_D + n\varphi f_X \right]^{-1}$$

$$= (1 + n\varphi) L \left( \frac{\theta m(\theta)}{s + \theta m(\theta)} \right) \left( 1 - \beta \right) \left( \frac{1 - \delta}{1 + r} \right) \left( \frac{\tilde{\varphi}}{\left( \frac{r + \delta}{1 + r} \right) \frac{f_X}{1 - G(\varphi_D^*)} + f_D + n\varphi f_X} \right).$$

**Proof of Proposition 2(i).** The definition of $\tilde{\varphi}$ in equation (11) and the equilibrium condition (16) imply that

$$\tilde{\varphi} = \varphi_D^* \left\{ \frac{1}{1 + n\varphi} \left[ \left( \frac{\tilde{\varphi}_D}{\varphi_D} \right)^{\sigma-1} + n\varphi \frac{f_X}{f_D} \left( \frac{\tilde{\varphi}_X}{\varphi_X} \right)^{\sigma-1} \right] \right\}^{\frac{1}{1-\tau}} = \varphi_D^* \left[ \frac{f_X/f_D}{1 - G(\varphi_D^*)} \left( \frac{1 + \tau}{1 + \tau + \varphi} \right) + 1 + n\varphi f_X f_D \right]^{\frac{1}{1-\tau}}.$$

(24)
As explained in Melitz, trade liberalization always raises \( \varphi^*_D \) as it shifts up the Zero Cutoff Profit condition but leaves the Free Entry condition unchanged. Hence \( \partial \left( \frac{1 + n \varphi}{1 + n \rho} \right) / \partial n \geq 0 \Rightarrow \vartheta \partial \varphi / \partial n \geq 0 \), which obviously holds true when \( f_X \geq f_D \). A similar result can be derived for \( \tau \) noticing that \( \partial \vartheta / \partial \tau < 0 \). On the other hand, \( f_X \) has two opposite effects: it reduces the share of exporting firms \( \varrho \) and it increases the ratio \( \{ f_X / f_D \} \). Thus, even when \( f_X > f_D \), the effect of \( f_X \) is \textit{a priori} ambiguous. The effect of trade liberalization on unemployment stated in Proposition 2(i) immediately follows from Corollaries 1 and 2. Given that the Wage curve is increasing in \( \theta \), it also follows that real wages are increasing in \( n \) and \( \tau \).

**Proof of Proposition 2(ii).** Because there does not exist a general closed-form solution for \( \varphi^*_D \), we have to impose a particular functional form on \( g(\varphi) \) in order to derive necessary conditions which are functions of the exogenous parameters. We follow the common practice in the literature by considering that \( g(\varphi) \) is Pareto, so that \( g(\varphi) = \frac{\tau}{\varphi} \left( \frac{\varphi}{X} \right)^\gamma \). Since the absolute value of \( \varphi \) is meaningless in our model, we can normalize \( \tilde{\varphi} \) to one without loss of generality. Then it holds true that

\[
\tilde{\varphi}_D = \left( \frac{\gamma}{1 - \sigma + \gamma} \right)^{\frac{1}{\gamma - 1}} \varphi^*_D \quad \text{and} \quad \tilde{\varphi}_X = \left( \frac{\gamma}{1 - \sigma + \gamma} \right)^{\frac{1}{\gamma - 1}} \varphi^*_X ,
\]

and \( \tilde{\varphi} \) can be decomposed as follows

\[
\tilde{\varphi} = \left[ \frac{1}{1 + n \varrho} \left( \tilde{\varphi}^{\sigma - 1}_D + n \varrho \tilde{\varphi}^{\sigma - 1}_X \right) \right]^{\frac{1}{\gamma - 1}} = \left( \frac{\gamma}{1 - \sigma + \gamma} \right)^{\frac{1}{\gamma - 1}} \varphi^*_D \left[ \frac{1 + n \varrho \left( f_X / f_D \right)}{1 + n \varrho} \right]^{\frac{1}{\gamma - 1}} .
\]

We can now use the equilibrium condition (16) to express \( \varphi^*_D \) as a function of the parameters

\[
\varphi^*_D = \left[ \left( \frac{f_D}{f_E} \right) \left( \frac{1 + \varrho}{\varrho} \right) \left( f_D \left( \frac{\tilde{\varphi}_D}{{\tilde{\varphi}_D}^{\gamma - 1}} \right) - 1 \right) + n \varrho f_X \left( \frac{\tilde{\varphi}_X}{{\tilde{\varphi}_X}^{\gamma - 1}} - 1 \right) \right]^{\frac{1}{\gamma - 1}} \]

\[
= \left[ \left( \frac{f_D}{f_E} \right) \left( \frac{1 + \varrho}{\varrho} \right) \left( \frac{\gamma}{1 - \sigma + \gamma} - 1 \right) \left( 1 + n \varrho \left( \frac{f_X}{f_D} \right) \right) \right]^{\frac{1}{\gamma - 1}} .
\]

Reinserting this expression into (25) and using the fact that \( \varrho = \tau^{-\gamma} (f_D / f_X)^{\frac{\gamma - 1}{\gamma}} \), we finally obtain

\[
\tilde{\varphi} = K_0 \left( 1 + n \tau^{-\gamma} (f_D / f_X)^{\frac{\gamma - 1}{\gamma}} \right)^{\frac{1}{\gamma - 1}} \frac{1 + n \tau^{-\gamma} (f_D / f_X)^{\frac{\gamma - 1}{\gamma}}}{\left( 1 + n \tau^{-\gamma} (f_D / f_X)^{\frac{\gamma - 1}{\gamma}} \right)} ,
\]

where

\[
K_0 \equiv \left( \frac{\gamma}{1 - \sigma + \gamma} \right)^{\frac{1}{\gamma - 1}} \left( \frac{f_D}{f_E} \right) \left( \frac{1 + \varrho}{\varrho} \right) \left( \frac{\gamma - 1}{1 - \sigma + \gamma} \right)^{\frac{1}{\gamma - 1}} .
\]

Differentiating this expression with respect to \( n \) shows that

\[
\frac{\partial \tilde{\varphi}}{\partial n} \geq 0 \Leftrightarrow \frac{\sigma - 1}{\gamma} \left( 1 + n \tau^{-\gamma} (f_D / f_X)^{\frac{\gamma - 1}{\gamma}} \right) \geq f_D / f_X - 1 .
\]
Since $f_X \tau \sigma^{-1} > f_D$, it is easily seen that: $\frac{\sigma^{-1}}{\gamma} (1 + n) \leq \frac{f_D}{f_X} - 1 \Rightarrow \partial \tilde{\varphi} / \partial n < 0$. This establishes that the necessary condition above can be violated. The effects of $\tau$ is easily derived following similar steps. Regarding the comparative static with respect to $f_X$, we have

$$\frac{\partial \tilde{\varphi}}{\partial f_X} \leq 0 \Leftrightarrow \left( \frac{\gamma}{\sigma - 1} \right)^2 \geq \frac{f_X}{f_X - f_D} \left[ 1 + n \tau^{-\gamma} \left( \frac{f_X}{f_D} \right)^{\frac{\gamma}{\sigma - 1}} \right]$$

Clearly, that inequality can hold only if $f_X > f_D$. The impact on employment and real wages is obtained from the same reasoning than in the proof of Proposition 2(i).

### B  Additional results

#### B.1 Collective bargaining

In a firm covered by collective bargaining, workers form a firm-wide coalition, that is, a trade union. When bargaining fails and workers go on strike, the firm loses not only the value associated to the marginal worker, as with individual bargaining, but its entire labor force. We opt for an efficient bargaining setup so that the firm and the union bargain about both wages and employment. This ensures that we are considering equilibria lying on the Pareto frontier.

34 Negotiations between the union and the firm take place in the first period. The union’s objective is the expected sum of its members’ rents

$$U(l, w) \equiv (1 - \delta) l \left[ \frac{w - rU_r + \delta}{r + \delta} \right],$$

while the firm seeks to maximize its expected variable profits

$$F(l, w; \varphi) \equiv \left( \frac{1 - \delta}{r + \delta} \right) \left[ R(l; \varphi) - wl(\varphi) - \frac{c}{m(\theta)} \chi l(\varphi) \right] - \frac{c}{m(\theta)} l.$$  

The negotiation specifies both employment and wages. The solution lies on the contract curve which connects the points where the firm iso-profit curves are tangent to the union indifference curves. The main results also hold in a right to manage setup where unions negotiate only about wages and firms have full freedom to set the level of employment. Barth and Zweimüller (1995) study different wage bargaining scenarios when firms are heterogeneous with respect to their productivity.

35 One could instead consider that the firm and the union bargain on the steady-state profits, so that $F(l, w; \varphi) = \left( \frac{1 + \delta}{1 + \delta} \right) \left[ R(l; \varphi) - wl(\varphi) - \frac{c}{m(\theta)} \chi l(\varphi) \right]$. This obviously generates a hold-up problem where the union does not take into account the initial recruitment costs. Then employment is lower and wages higher but the main insights are not fundamentally modified.
actual agreement is pinned down by the union’s bargaining power $\beta$. Proposition 2 shows that the labor market equilibrium can be characterized in a similar fashion than in the Individual Bargaining regime.

**Proposition 3** When wages are collectively bargained, the labor market admits an equilibrium if and only if $b < (\sigma - 1)/\sigma$. The equilibrium is unique and such that wages are constant across firms. The equilibrium wage, $w$, and vacancy-unemployment ratio, $\theta$, simultaneously satisfy the following Wage and Labor Demand conditions:

$$W: \quad w = \frac{\beta}{\sigma(1 - b)} \left( \frac{\theta m(\theta)}{r + s} \right) \tilde{\varphi} + \frac{\beta}{\sigma(1 - b)} \tilde{\psi}$$

$$LD: \quad w = \left( 1 - \frac{1 - \beta}{\sigma} \right) \tilde{\varphi} - \frac{c}{m(\theta)} \left( \frac{r + s}{1 - \delta} \right)$$

**Proof of Proposition 3.** The contract curve is given by the points where the firm iso-profit curves are tangent to the union’s indifference curves, so that

$$\frac{\partial F(l, w; \varphi)}{\partial l} = \frac{\partial U(l, w; \varphi)}{\partial l} \Rightarrow \frac{\partial R(l; \varphi)}{\partial l} = rU + \left( \frac{r + s}{1 - \delta} \right) \frac{c}{m(\theta)}.$$  \hspace{2cm} (30)

The actual contract solves the following Nash-bargaining problem $^{36}$

$$\max_{w,l} \Omega(w, l; \varphi) \equiv U(l, w; \varphi)^{\beta} F(l, w; \varphi)^{1-\beta}.$$  \hspace{2cm} (31)

The union and the firm split the forward looking surplus. $^{37}$ The first order condition with respect to the wage rate is

$$w(\varphi, l) = (1 - \beta) rU + \beta \left[ \frac{R(l; \varphi)}{l} - \left( \frac{r + s}{1 - \delta} \right) \frac{c}{m(\theta)} \right] = rU + \left( \frac{\beta}{\sigma} \right) \frac{R(l; \varphi)}{l},$$

where the second equality is obtained substituting the Pareto optimality condition (30) and using the identity $\partial R(l; \varphi)/\partial l = \left( \frac{r + s}{\sigma} \right) R(l; \varphi)/l$. Equation (32) is the Wage curve under collective bargaining. The Labor Demand curve is given by the first order condition of problem (31) with respect to the employment level

$$w(\varphi, l) = \left( 1 - \frac{1 - \beta}{\sigma} \right) \frac{R(l; \varphi)}{l} - \left( \frac{r + s}{1 - \delta} \right) \frac{c}{m(\theta)}$$

$^{36}$The setup cost is sunk and so cannot be recovered by the firm in case of disagreement with the union. Thus it does not enter the firm outside option. If one assume, as in Melitz (2003), that operating costs are paid in each period, the strategic form of the Nash-bargaining problem still holds as long as the firm cannot default on his payment following a breakdown in the wage negotiation. Notice, however, that when fixed costs are included in the firm threat point, the solution to (31) does not lie on the contract curve and so violates the axiom of Pareto optimality. Hence, our formulation can also be justified on axiomatic ground.

$^{37}$Considering instead that disagreement delays production does not fundamentally affect our result.
Both conditions indicate that wages are identical across firms since, as explained in the proof of Proposition 1, $R(l; \varphi) / l = p_D(\varphi) \varphi = p_D(\tilde{\varphi}) \tilde{\varphi} = \tilde{\varphi}$. The employees’ outside option therefore reads

$$r_U(\theta) = bw + \theta m(\theta) \left( \frac{w - r_U}{r + s} \right) = bw + \theta m(\theta) \left( \frac{\beta}{\sigma(r + s)} \right) \tilde{\varphi},$$

(34)

where the last equality follows from (32). Combining the three equations above, yields the expressions in Proposition 3. The existence and uniqueness requirements follow from the same reasoning than in the proof of Proposition 1.

For the same reasons than before, the Wage curve is increasing in $\theta$ while the Labor Demand curve is decreasing. The bargained wage is equal to the opportunity cost of employment $r_UC$ plus a share $\beta$ of the remaining profits per worker. Due to the existence of rent-sharing, and in contrast to individual bargaining, the slope of the Wage curve is increasing in aggregate productivity. Yet, as with individual bargaining, the wage rate is the same across firms with different levels of productivity. Figure 2 in FPS (2008) illustrates why, in our CES setting, differences in idiosyncratic productivity wash out so that there is no wage dispersion.

The most significant difference with the individual bargaining regime is that now average productivity $\tilde{\varphi}$ also raises the slope of the Wage curve. Yet, as stated in Corollary 2, this additional effect on the Wage curve is again unambiguously dominated by the shift of the Labor Demand curve.

**Corollary 2** When wages are collectively bargained, the vacancy-unemployment ratio $\theta_C$ is increasing in aggregate productivity $\tilde{\varphi}$.

**Proof of Corollary 2.** The proof is established in a similar fashion as Corollary 1. Combining the Labor Demand and Wage curves in Proposition 3 leads to the following equilibrium requirement

$$\Psi (\theta; \tilde{\varphi}) \equiv \tilde{\varphi} \left( \frac{\beta}{\sigma(1 - b)} \left( \frac{\theta m(\theta) + r + s}{r + s} \right) + \frac{-\sigma + 1 - \beta}{\sigma} \right) + \left( \frac{r + s}{1 - \delta} \right) \frac{c}{m(\theta)} = 0.$$

Differentiating $\Psi (\theta; \tilde{\varphi})$ with respect to $\tilde{\varphi}$ and $\theta$ yields

$$\frac{\partial \theta}{\partial \tilde{\varphi}} = -\frac{\partial \Psi (\theta; \tilde{\varphi}) / \partial \tilde{\varphi}}{\partial \Psi (\theta; \tilde{\varphi}) / \partial \theta} > 0.$$

The inequality sign follows from

$$\frac{\partial \Psi (\theta; \tilde{\varphi})}{\partial \tilde{\varphi}} = \frac{\beta}{\sigma(1 - b)} \frac{\theta m(\theta) + r + s}{r + s} + \frac{-\sigma + 1 - \beta}{\sigma} = -\frac{1}{\tilde{\varphi}} \left( \frac{r + s}{1 - \delta} \right) \frac{c}{m(\theta)} < 0,$$

(35)

$$\frac{\partial \Psi (\theta; \tilde{\varphi})}{\partial \theta} = \tilde{\varphi} \beta \left[ \theta m'(\theta) + m(\theta) \right] - \left( \frac{r + s}{1 - \delta} \right) \frac{cm'(\theta)}{m(\theta)^2} > 0.$$

(36)
The last equality follows from \( \Psi(\theta; \bar{\varphi}) = 0 \) and the sign of the inequality holds true due to the homogeneity of degree one of the matching function. Finally, the LMC is given by

\[
LMC_C : M(\theta) = (1 + n\varrho)(1 - u(\theta)) L \left( \frac{1 - \beta}{\sigma} \right) \left( \frac{1 - \delta}{1 + r} \right) \left( \frac{\bar{\varphi}}{\frac{r + \delta}{1 + r} + \frac{F_E}{1 - G(\varphi_D)} + f_D + n\varrho f_X} \right). \tag{37}
\]

### B.2 Equilibrium with external economies of scale

Labor market tightness, real wages and input diversity are determined jointly when there are external economies of scale \((\nu > 0)\). Their equilibrium values follow from the Labor Demand, Wage Curve and Labor Market Clearing conditions, as defined in Sections 3 and 5.1. To clarify the analysis, we combine the Labor Demand and Wage Curve into one equation that we label \textit{Equilibrium Tightness Condition} (ETC). As the LMC, the ETC defines a mapping between input diversity \(M\) and labor market tightness \(\theta\).

We can then combine the LMC and the ETC for each bargaining environment to pin down the equilibrium values of \(M\) and \(\theta\). Index \(I\) denotes individual and \(C\) collective bargaining, respectively. Using (9) and (10) for individual, (29) and (28) for collective bargaining, we obtain

\[
ETC_I : M_I(\theta_I) = \left[ \frac{c}{(1 - \beta)(1 - \delta)} \left( \frac{1 - \delta}{m(\theta_I)} \right) + \beta\theta_I \right]^{\frac{\sigma - 1}{\sigma}}. \tag{38}
\]

\[
ETC_C : M_C(\theta_C) = \left[ \frac{c}{(1 - \delta)} \left( \frac{1 - \delta}{m(\theta_C)} \right) + \beta\theta_C \right]^{\frac{\sigma - 1}{\sigma}}. \tag{39}
\]

The ETCs are upward-sloping in each bargaining regime because more input diversity raises efficiency and thus compensates the increase in recruitment costs as \(\theta\) goes up. The LMC curves also need some adjustment since they also depend on the economies of scale parameter \(\nu\).

\[
LMC_I : M_I(\theta_I) = \left[ (1 + n\varrho)(1 - u(\theta_I)) L \left( \frac{1 - \beta}{\sigma - \beta} \right) \left( \frac{1 - \delta}{1 + r} \right) \left( \frac{\bar{\varphi}}{\frac{r + \delta}{1 + r} + \frac{F_E}{1 - G(\varphi_D)} + f_D + n\varrho f_X} \right) \right]^{\frac{\sigma - 1}{\sigma}}. \tag{40}
\]

For collective bargaining, the LMC is given by

\[
LMC_C : M_C(\theta_C) = \left[ (1 + n\varrho)(1 - u(\theta_C)) L \left( \frac{1 - \beta}{\sigma} \right) \left( \frac{1 - \delta}{1 + r} \right) \left( \frac{\bar{\varphi}}{\frac{r + \delta}{1 + r} + \frac{F_E}{1 - G(\varphi_D)} + f_D + n\varrho f_X} \right) \right]^{\frac{\sigma - 1}{\sigma}}. \tag{41}
\]
Given that the LMCs conditions are also increasing, equilibrium existence and uniqueness are not anymore ensured but can be established imposing empirically reasonable restrictions.\textsuperscript{38}

**Lemma 2** When $\nu \geq 0$, equilibrium tightness and input diversity are pinned down by the system \{(40), (38)\} for the case of individual bargaining and by \{(41), (39)\} for the case of collective bargaining. Assume that the aggregate matching function is Cobb-Douglas, so that $m(\theta) = m_0 \theta^{-\alpha}$, with $m_0 > 0$ and $\alpha \in (0, 1)$.

In case of individual bargaining, a sufficient condition for existence and uniqueness of an equilibrium with $u \in (0, 1)$ is $\nu/(\sigma - 1) < \alpha$. For the collective bargaining scenario, a sufficient condition is $\nu/(\sigma - 1) < \min \{\alpha, \frac{1}{2}\}$.

**Proof of Lemma 2**

**Individual Bargaining.** It is easily seen that, when $\sigma + (1 - \nu) > 2$, both (40) and (38) converge to zero as $\theta_I$ goes to zero. When $\theta_I$ goes to infinity, (38) diverges to infinity whereas (40) converges to

$$M \equiv \left[ L(1 + n\theta) \left( \frac{1 - \beta}{\sigma - \beta} \right) \left( \frac{1 - \delta}{1 + r} \right) \left( \frac{\tilde{\varphi}_I}{\tilde{\varphi}_I} \right) \left( \frac{1}{\nu K_2} \right) \left( f_D + n g f_X \right) \right]^{1 - \sigma} < \infty.$$

Hence, the existence of an equilibrium is established if the derivative of (38) at $\theta_I = 0$ is inferior to that of (40). Since

$$\frac{\partial M_I}{\partial \theta_I} \bigg|_{ETC_I} = \frac{\sigma - 1}{\nu} K_1 \left( \frac{(1 - b(1 - \beta))(r + s) + \beta \theta_I m(\theta_I)}{m(\theta_I)} \right)^{\frac{\sigma - 1}{\nu} - 1} \left( \frac{\alpha(1 - b(1 - \beta))(r + s) + \beta \theta_I m(\theta_I)}{\theta_I m(\theta_I)} \right),$$

with $K_1 \equiv c \left(1 - \beta\right)(1 - b)(1 - \delta)\tilde{\varphi} \left(\frac{\sigma - 1}{\sigma - \beta}\right)^{-1}$, the derivative of (40) w.r.t. $\theta_I$ converges to zero as $\theta_I$ goes to zero if and only if: $\lim_{\theta_I \to 0} \theta_I m(\theta_I)^{\frac{\sigma - 1}{\nu}} = \infty$. With the Cobb-Douglas specification, this requirement is fulfilled when $\alpha > \frac{\nu}{\sigma - 1}$. Consider now the derivative of (40) w.r.t. $\theta_I$

$$\frac{\partial M_I}{\partial \theta_I} \bigg|_{LMC_I} = \frac{\sigma - 1}{\sigma - 1 - \nu} K_2 \left( \frac{\theta_I m(\theta_I)}{s + \theta_I m(\theta_I)} \right)^{\frac{1}{\nu K_2} - 1} \left( \frac{(1 - \alpha)m(\theta_I)s}{(s + \theta_I m(\theta_I))^2} \right),$$

where

$$K_2 \equiv \left[ L(1 + n\theta) \left( \frac{1 - \beta}{\sigma - \beta} \right) \left( \frac{1 - \delta}{1 + r} \right) \left( \frac{\tilde{\varphi}_I}{\tilde{\varphi}_I} \right) \left( \frac{1}{\nu K_2} \right) \left( f_D + n g f_X \right) \right]^{1 - \sigma}.$$

\textsuperscript{38}As explained in subsection 5.1, when $\sigma < \nu + 1$, the LMCs conditions are decreasing in $\theta$. Hence, there always exists a unique equilibrium when this parameter restriction is satisfied. Yet, we do not focus on this case because it is neither theoretically realistic nor empirically relevant.
Hence it diverges to infinity as \( \theta \) goes to zero if and only if: \( \lim_{\theta \to 0} \theta^{1-\sigma \alpha^{-\sigma}} \equiv m(\theta)^{1-\sigma \alpha^{-\sigma}} = \infty \). With the Cobb-Douglas specification, this requirement is fulfilled when \( \alpha > \frac{1}{\sigma - 1} \). Hence, equilibrium existence is established.

The uniqueness of the equilibrium follows from the fact that (38) is convex while (40) is concave in \( \theta_I \). Since

\[
\frac{\partial^2 M_I}{\partial \theta^2}_{|ETC_I} = K_1 \left( \frac{\sigma - 1}{\nu} \right) [Z_1 - Z_2],
\]

where

\[
Z_1 = \left( \frac{\sigma - 1}{\nu} - 1 \right) (H(r + s)m_0^{-1}\theta_I^{\alpha - 1} + \beta \theta_I)^{\frac{\sigma - 1}{\nu} - 2} (\alpha H(r + s) m_0^{-1}\theta_I^{\alpha - 1} + \beta)^2,
\]

\[
Z_2 = (1 - \alpha) (H(r + s)m_0^{-1}\theta_I^{\alpha - 1} + \beta \theta_I)^{\frac{\sigma - 1}{\nu} - 1} \alpha H(r + s) m_0^{-1}\theta_I^{\alpha - 2}.
\]

and \( H = (1 - b(1 - \beta)) \). The second derivative of (38) is positive when

\[
Z_1 > Z_2 \iff \left( \frac{\sigma - 1}{\nu} - 1 \right) (\alpha H(r + s)m_0^{-1}\theta_I^{\alpha - 1} + \beta)^2 > (1 - \alpha) (H(r + s)m_0^{-1}\theta_I^{\alpha} + \beta \theta_I) \alpha H(r + s)m_0^{-1}\theta_I^{\alpha - 2}.
\]

But the term on the left-hand side of the inequality can be lower-bounded as follows

\[
\left( \frac{\sigma - 1}{\nu} - 1 \right) (\alpha H(r + s)m_0^{-1}\theta_I^{\alpha - 1} + \beta)^2 > \left( \frac{\sigma - 1}{\nu} - 1 \right) \alpha^2 (H(r + s)m_0^{-1}\theta_I^{\alpha - 1} + \beta)^2>
\]

\[
> \left( \frac{\sigma - 1}{\nu} - 1 \right) \alpha H(r + s)m_0^{-1}\theta_I^{\alpha} + \beta \theta_I \alpha H(r + s)m_0^{-1}\theta_I^{\alpha - 2}.
\]

Thus (38) is convex when \( \left( \frac{\sigma - 1}{\nu} - 1 \right) > 1 - \alpha \iff \alpha > \frac{\nu}{\sigma - 1} \). Similarly differentiating twice (40) w.r.t. \( \theta_I \) yields

\[
\left. \frac{\partial^2 M_I}{\partial \theta^2} \right|_{LMC_I} = \frac{K_2 (1 - \alpha) \theta_I^{1-\alpha} - \theta_I^{(1-\alpha)\theta_I^{1-\alpha}}}{(s + m_0 \theta_I^{1-\alpha})^2 \left( \frac{\sigma - 1}{\nu} - 1 \right) [Z_3 - Z_4]},
\]

where

\[
Z_3 = \left( \frac{1 - \sigma}{1 - \sigma + \nu} - 1 \right) m_0^{-1}\theta_I^{1-\alpha} \left[ \frac{1 - \sigma}{1 - \sigma + \nu} \theta_I^{1-\alpha} \right]^{\frac{\sigma - 1}{\nu} - 2} (s + m_0 \theta_I^{1-\alpha})^{\frac{\sigma - 1}{\nu} - 1},
\]

\[
Z_4 = \left( \frac{1 - \sigma}{1 - \sigma + \nu} + 1 \right) (1 - \alpha) m_0^{-1}\theta_I^{1-\alpha} \left[ \frac{1 - \sigma}{1 - \sigma + \nu} \theta_I^{1-\alpha} \right]^{\frac{\sigma - 1}{\nu} - 1} (s + m_0 \theta_I^{1-\alpha})^{\frac{\sigma - 1}{\nu} - 1},
\]

which is negative when \( \left( \frac{1 - \sigma}{1 - \sigma + \nu} \right) < 1 \iff \alpha > \frac{\nu}{\sigma - 1} \).

**Collective Bargaining.** The \( ETC_C \) is well defined only if

\[
\left. \frac{\sigma - 1}{\sigma} - \frac{\beta b}{\sigma (1 - b)} \right|_{\frac{\theta C M (\theta_C)}{r + s}} > \frac{\beta b}{\sigma (1 - b)} \frac{\theta C M (\theta_C)}{r + s} \iff \frac{\sigma - 1}{\sigma} = \frac{\beta b}{\sigma (1 - b)} = \frac{\beta b}{\sigma (1 - b)} \frac{\theta C M (\theta_C)}{r + s} = \frac{\beta b}{\sigma (1 - b)} \frac{\theta C M (\theta_C)}{r + s},
\]

which makes sure that the equilibrium mass is real-valued and from where it follows that \( \chi > 1 \). This restriction allows to find a sufficient condition for the strict convexity of \( ETC_C \), very much in line with
the proof for Individual bargaining.

It is easily seen that \((40)\) and \((38)\) converge to zero as \(\theta\) goes to zero. When \(\theta\) goes to the upper bound \(\bar{\theta}_C\), \((39)\) diverges to infinity whereas \((41)\) converges to some \(\bar{M}_C < \infty\). Under collective bargaining the first derivative of the \(ETC\) with respect to \(\theta_C\) is

\[
\frac{\partial M_C}{\partial \theta_C} \bigg|_{ETC} = \frac{\sigma - 1}{\nu} \frac{1}{\varphi^{\frac{\tau - 1}{\nu}} (\alpha - \frac{\beta b}{\nu(1 - b)} - \frac{\beta \theta_C^\alpha}{\varphi(1 - b)} - \frac{m_0 \theta_C^\alpha}{\tau + s})^{\frac{\tau - 1}{\nu}} + 1}
\]

where

\[
Z_5 = \left\{ \alpha \left[ \left( \frac{\sigma - 1}{\nu} - \frac{\beta b}{\sigma(1 - b)} \right) \theta_C^\alpha \left( \frac{\nu}{\sigma(1 - b)} - \frac{m_0 \theta_C^\alpha}{\tau + s} \right) \right] \right\},
\]

\[
Z_6 = \left\{ (1 - \alpha) \frac{\beta m_0 \theta_C^\alpha}{\sigma(1 - b) r + s} \right\}.
\]

which converges to zero when \(\theta\) goes to zero if \(\alpha > \frac{\nu}{\tau + 1}\). The slope of \(LMC\) in \(\theta_C\) depends on the same conditions than \(LMC_I\). The \(LMC_C\) is strictly concave (the proof is identical to the case of individual bargaining). The strict concavity of \(ETC\) requires

\[
\frac{\partial^2 M_C}{\partial \theta_C^2} \bigg|_{ETC} = \frac{\sigma - 1}{\nu} \frac{1}{\varphi^{\frac{\tau - 1}{\nu}} (\alpha - \frac{\beta b}{\nu(1 - b)} - \frac{\beta \theta_C^\alpha}{\varphi(1 - b)} - \frac{m_0 \theta_C^\alpha}{\tau + s})^{\frac{\tau - 1}{\nu}} + 1}
\]

where \(Z_7 = \left\{ \left( \frac{\nu}{\sigma(1 - b)} - \frac{\beta \theta_C^\alpha}{\varphi(1 - b)} - \frac{m_0 \theta_C^\alpha}{\tau + s} \right)^{\frac{\tau - 1}{\nu}} \right\}\) and

\[
Z'_5 = \left\{ \alpha \left[ \left( \frac{\sigma - 1}{\nu} - 1 \right) \left( \frac{\sigma - 1}{\sigma(1 - b)} - \frac{\beta b}{\sigma(1 - b)} \right) \theta_C^\alpha \left( \frac{\nu}{\sigma(1 - b)} - \frac{m_0 \theta_C^\alpha}{\tau + s} \right) \right] \right\},
\]

\[
Z'_6 = \left\{ (1 - \alpha) \frac{\beta m_0 \theta_C^\alpha}{\sigma(1 - b) r + s} \right\},
\]

\[
Z'_7 = - (1 - \alpha) \left( \frac{\sigma - 1}{\nu} + 1 \right) \left( \frac{\sigma - 1}{\sigma} - \frac{\beta b}{\sigma(1 - b)} - \frac{\beta m_0 \theta_C^\alpha}{\sigma(1 - b) r + s} \right) \left( \frac{\beta m_0 \theta_C^\alpha}{\sigma(1 - b) r + s} \right).
\]

The second derivative is positive when \(Z'_5 + Z'_6\) is positive which holds true if

\[
\alpha \left( \frac{\sigma - 1}{\nu} - 1 \right) \left( \frac{\sigma - 1}{\sigma(1 - b)} - \frac{\beta b}{\sigma(1 - b)} \right) \theta_C^\alpha \left( \frac{\nu}{\sigma(1 - b)} - \frac{m_0 \theta_C^\alpha}{\tau + s} \right) + (1 - 2 \alpha) \alpha \left( \frac{\sigma - 1}{\nu} - 1 \right) \beta m_0 \theta_C^\alpha > 0
\]

while \(\alpha > \frac{\nu}{\tau + 1}\) and \(\sigma > 2\). This condition can be lower bounded using \((46)\). Thus we know that the second derivative is positive as long as the following condition holds

\[
\alpha \left( \frac{\sigma - 1}{\nu} - 1 \right) \nu + (1 - 2 \alpha) \alpha \left( \frac{\sigma - 1}{\nu} - 1 \right) > 0
\]

Let \(\Upsilon \to 1\), then \((\sigma - 1)/\nu > 2\) secures that the the second derivative is positive. Hence, the equilibrium is unique if \(\frac{\nu}{\sigma - 1} < \min \left\{ \alpha, \frac{1}{2} \right\}\).
Figure 4: Determination of input diversity and labor market tightness in general equilibrium with \( \nu > 0 \).

Figure 4 shows the equilibrium conditions when wages are bargained at the individual level. Under the parameter restrictions presented in Lemma 2, both the ETC and the LMC start at the origin. The ETC is strictly convex while the LMC is strictly concave over the relevant parameter ranges. The LMC converges to some upper bound on input diversity \( \bar{M} \) while the ETC diverges. The collective bargaining case looks almost identical.\(^{39}\) Hence, the existence of a unique equilibrium (point \( E \)) is guaranteed. As \( \nu \to 0 \), the ETC locus converges towards a vertical line, whose position is pinned down by average productivity \( \bar{\varphi} \) and labor market variables.\(^{40}\)

The parameter restriction stated in Lemma 2 requires that the strength of the external scale effect is sufficiently low when compared to the elasticity of the matching function \( \alpha \). Empirically, sectoral estimates of \( \nu \) and \( \alpha \) cluster around 0.5,\(^{41}\) hence \( \sigma \) would need to be above 2. This requirement does not

\(^{39}\) The only difference is that the ETC locus asymptotes towards some tightness \( \bar{\theta} > 0 \) implicitly determined by
\[ \frac{\partial m(\bar{\theta})}{\partial \bar{\theta}} = \frac{3-1}{\sigma} - b > 0. \]

\(^{40}\) When the flow-value of non market activity is not indexed to aggregate productivity but instead equal to an exogenous constant, the ETC locus has a positive intercept on the vertical axis. As can easily be seen from Figure 4, this implies that the model admits at least two equilibria or none. This explains why we have assumed from the outset that non market activity yields revenues proportional to \( \Phi \). Janiak (2006) considers instead that they are purely exogenous and so, in order to circumvent the multiplicity issue, focuses on cases where \( \sigma < \nu + 1 \). This is also why he finds a positive relationship between variable trade costs and employment.

\(^{41}\) See Ardelean (2007) for estimates of the external scale effect and Pissarides and Petrongolo (2001) for estimates of the matching function parameters.
seem implausible empirically.

Only if \( \nu > 0 \) do changes in labor market parameters affect aggregate productivity \( \Phi \). Average productivity of input producer, however, remains unchanged, as Lemma 1 still applies. Hence, labor market institutions matter for aggregate productivity only through their effect on input diversity. Inspection of the equilibrium conditions reveals that higher levels of \( b \) or \( c \) rotate the ETC loci upwards, while they do not affect the LMC curves. In both bargaining regimes, those changes lower labor market tightness, real wages, and increase unemployment. In contrast, an improvement in the matching efficiency \( m_0 \) affects the ETC and the LMC curves. The LMC loci rotate upwards, while the ETC curves move in opposite direction: equilibrium tightness unambiguously increases, leading to higher real wages and lower unemployment.

To sum up, labor market parameters have a qualitatively similar impact on unemployment than in the standard Pissarides (2000) model with homogeneous firms and perfect competition on product markets. Notice also that economies of scale generate a negative relationship between the external size of the economy, \( L \), and the rate of unemployment. Given that such a correlation is not substantiated by the data, the model suggests that the marginal scale effect has to be small, either because of a low value of \( \nu \) or a very high degree of input diversity.

B.2.1 Trade liberalization and unemployment with external economies of scale

We are now able to characterize the effect of trade liberalization on labor market outcomes when the production function exhibits external economies of scale. This is done in the next proposition, which – as Proposition 2 – falls in two parts. Part (i) provides a sufficient condition for some trade liberalization scenarios to improve labor market outcomes.\(^{42}\) Part (ii) assumes that the productivity distribution is Pareto and provides necessary and sufficient conditions.

**Proposition 4** Assume that there are external economies of scale \( (\nu > 0) \) and that the existence and uniqueness condition in Lemma 2 is satisfied.

(i) If \( f_X \geq f_D \) a reduction of variable trade costs \( \tau \) or an increase in the number of trading partners \( n \) lead to a fall in the equilibrium rate of unemployment and a rise in the real wage, regardless of whether wages are bargained individually or collectively. A fall in fixed foreign distribution costs has an ambiguous effect on labor market outcomes.

\(^{42}\)Baldwin and Forslid (2006) provide conditions for different globalization scenarios to improve average productivity in a Melitz model with \( \nu = 1 \) and \( \varphi \) following the Pareto distribution.
(ii) Let firms draw their productivities from a Pareto distribution. Then, regardless of the wage bargaining regime, the equilibrium rate of unemployment falls and the real wage rises (a) due to a reduction in $\tau$ or an increase in $n$ if and only if
\[
\frac{\sigma - 1}{\gamma (1 - \nu)} \left( 1 + n\tau^{-\gamma} \left( \frac{f_D}{f_X} \right)^{\frac{\gamma}{\gamma - 1}} \right) \geq \frac{f_D}{f_X} - 1,
\]
(b) due to a reduction in $f_X$ if and only if
\[
\frac{\gamma^2 (1 - \nu)}{(\sigma - 1)(\sigma - 1 - \nu\gamma)} \geq \frac{f_X}{f_X - f_D} \left[ 1 + n\tau^{-\gamma} \left( \frac{f_D}{f_X} \right)^{\frac{\gamma}{\gamma - 1}} \right].
\]

**Proof of Proposition 4(ii)** When firms draw their productivities from a Pareto distribution, we can use (26) to substitute $\varphi_D$ in condition (49). Since $\varphi$ can be replaced by $\tau^{-\gamma} (f_D/f_X)^{\frac{\gamma}{\gamma - 1}}$, we obtain
\[
sign \left\{ \frac{\partial \theta}{\partial n} \right\} = sign \left\{ \frac{\partial}{\partial n} \left[ (1 + n\tau^{-\gamma} (f_D/f_X)^{\frac{\gamma}{\gamma - 1}})^{-\lambda_2} \left( 1 + n\tau^{-\gamma} (f_D/f_X)^{\frac{\gamma}{\gamma - 1}} \right)^{\frac{\gamma - 1}{\gamma}} \right] \right\}.
\]

Differentiating this expression with respect to $n$ shows that
\[
\frac{\partial \theta}{\partial n} \geq 0 \iff \left( \frac{\sigma - 1}{\gamma (1 - \nu)} \right) \left( 1 + n\tau^{-\gamma} (f_D/f_X)^{\frac{\gamma}{\gamma - 1}} \right) \geq \frac{f_D}{f_X} - 1.
\]
Obviously, this condition is always satisfied when $(1 - \nu) = 0$, as in Melitz (2003). A similar result follows for $\tau$. An increase in foreign beachhead costs leads to a decreasing average productivity if the following requirement is fulfilled
\[
\frac{\partial \theta}{\partial f_X} \leq 0 \iff \frac{\gamma^2 (1 - \nu)}{(\sigma - 1)(\sigma - 1 - \nu\gamma)} \geq \frac{f_X}{f_X - f_D} \left[ 1 + n\tau^{-\gamma} \left( \frac{f_D}{f_X} \right)^{\frac{\gamma}{\gamma - 1}} \right].
\]
Proposition 4 generalizes Proposition 2 to cases with external economies of scale, as long as the additional parameter restriction ensuring existence and uniqueness of the equilibrium is satisfied. As discussed before, the requirements in Proposition 4 are largely satisfied by empirically reasonable parameter values. Accordingly, our theoretical analysis leads us to the conclusion that trade openness is likely to have a beneficial impact on labor market outcomes.

Figure 4 illustrates our findings: when $\tilde{\varphi}$ goes up, the ETC locus rotates downwards; the effect on the LMC curve, however, depends on parameters. Nevertheless, even when the LMC locus rotates down, the net effect on $\theta$ is positive in both wage bargaining scenarios. The effect on input diversity, in contrast, remains ambiguous.

Part (ii) of Proposition 4 derives sufficient and necessary conditions under the Pareto assumption. Inspection of condition (a) shows that the higher external economies of scale are, the more likely it is that labor market outcomes are improved by a reduction in export tariffs or an increase in the number of trading partners. Accordingly, when economies of scale are maximal ($\nu = 1$), as in Melitz (2003), condition (a) is always satisfied. The influence of $\nu$ is rather intuitive: trade raises not only productivity at the factory gate but also input diversity and this second effect is obviously more beneficial when economies of scales are strong.
References


